A Plain and Comprehensive

TREATISE
OF
Decimals and Menfuration.

Calculated for the INSTRUCTION and IMPROVEMENT of thofe who are defirous to be acquainted with FIGURES, in accurate and ufeful COMPUTATIONS.

CONTAINING,

I. AN Application of Decimals to all the leading Branches of ARITHMETIC, in a Variety of Examples; together with the Doctrine of Circulating Decimals or Repetends.

II. INTEREST in all its Cafes, both Simple and Compound; with Conftructions upon the Rules of Rebate or Difcount: Likewife feveral new Methods adapted to the Valuing of Leafes or Annuities on Lives.

III. THE Extraction and Ufes of the Square and Cube Roots, particularly a new, fhort, and eafy Method of Extracting the Cube Root.

IV. DUODECIMALS, or Crofs-Multiplication, by an eafier Method than hitherto practifed.

V. MENSURATION of all Kinds of Superficies and Solids, with practical Errors pointed out; together with the Art of Gaging, and fome improv'd Methods in Surveying of Land: Alfo the various Ufes of Specific Gravity.

And to accommodate the above WORK, is added,

VI. AN Introduction to practical Mufic; together with the Conftruction of Organ Pipes, Spinets, &c.

The Whole rendered familiar to the meaneft Capicity.

By WM. CRUMPTON.

BIRMINGHAM: Printed for the EDITOR; and fold by *Z. Stuart*, at the *Lamb*, in *Pater nofter-Row*, *London*, and moft other Bookfellers in Town and Country.

[*Price bound Three Shillings.*] 1776?

Dup 90

A Plain and Comprehensive

TREATISE

OF

Decimals and Mensuration.

Calculated for the

Instruction and Improvement of those Who are desirous to be acquainted with FIGURES, in accurate and useful Computations.

CONTAINING

I. An Application of Decimals to the [illegible] ARITHMETICK, in a Variety of Examples; together with the Doctrine of [illegible] Decimals.

II. [illegible] Gauging [illegible] Cask [illegible] with Contractions upon the Rules [illegible] Dimensions [illegible] several new [illegible] the [illegible]

III. [illegible] Superficial and Solid [illegible] the Square and Cube Roots [illegible] and [illegible]

IV. [illegible] Mensuration, by an easier Method than [illegible]

V. [illegible] with practical [illegible] the [illegible] of [illegible] and [illegible] Method [illegible] Land. [illegible]

And to accommodate the above Works, is added,

VI. An Introduction to [illegible] together with the [illegible] of [illegible] Figures, &c.

The Whole [illegible]

By Wm. CRUMPTON.

[illegible] Printed for the [illegible] and [illegible] at [illegible]

The Editor *to the* Public.

AMONG the many improvements of late years that have been made in arithmetic, the invention of DECIMALS is the moſt prevalent of any, for its diſpatch in numbers, and accuracy in computation. Its utility in matters of buſineſs and concern has induced me to put forth a Tract to endeavour to render DECIMALS more familiar than has hitherto been; including ſeveral uſeful matters that are new and conciſe, and which are laid down in the plaineſt manner, with the work to each example.

IN matters of this kind an Editor can't be too careful in the circumſpection of every article he inſerts; as alſo in reviſing the ſame; yet after all, there are very few performances that can pleaſe every taſte; for ſome are too apt to cavil if they do but meet with things out of their own way, while others are as much remiſs in furniſhing the mind with any kind of improvement whatever: But learning creates a beneficial notion of the moſt intereſting concerns of life, and forms a man for ſociety.

MY

My deſign in introducing Practical Muſic, is conſidered with this view, that it may be a means of relaxation at any time when the mind is rather wearied with too cloſe an application to ſtudy or buſineſs.

But the chief inducement of the whole undertaking has been more for the public good than any lucrative views of my own; therefore if any ſubject matter ſhould appear not couched in the moſt engaging terms, it is hoped all cenſure on that head may be ſilent, as this piece was wholly deſigned for general uſe.

The Subſcription I have met with in my undertaking, together with the ſanction of ſeveral learned and ingenious Gentlemen, are motives which have far exceeded my moſt ſanguine hopes: Therefore 'tis my ardent wiſh, that the learner may meet with the deſired ſatisfaction in the following pages.

A PLAIN and COMPREHENSIVE

TREATISE OF DECIMALS AND MENSURATION.

Decimal Arithmetic.

DECIMALS were invented for the easy working of fractional parts, by a much more commodious and compendious method than that of Vulgar Fractions, and is therefore now become of general uſe in all the branches of arithmetic; for the ſeveral operations perform'd in decimals, differ but little from thoſe of whole numbers.

A fraction whether vulgar or decimal is ſome part or parts of a unit. To expreſs it as a vul-

gar fraction it is written down as it comes from division; thus, if 37 was to be divided by 5, the quotient wou'd be more than 7, and less than 8; that is, it would be $7\frac{2}{5}$, or 7 times and 2 parts of 5: Here the uppermost number of the fraction is call'd the Numerator, and that under the line is the Denominator or whole number.

Now decimal fractions are wrote without their denominators, for the integer or whole thing being always understood to consist of an unit with as many cyphers annex'd to the right hand, as the decimal fraction has places; and here the numerator or decimal itself is distinguish'd from whole numbers by a separating point; thus, 5.4 is $5\frac{4}{10}$; and 0.7 is $\frac{7}{10}$; 35.05 is $35\frac{5}{100}$, &c. But the different value of the several places will more plainly appear from the following Table.

Whole numbers. Decimal parts.

	6	5	4	3	2	1	.1	2	3	4	5	6	
&c.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units place.	Parts of Ten, or $\frac{1}{10}$.	Parts of a Hundred.	Parts of a Thousand.	Parts of Ten Thousand.	Parts of a 100 Thousand.	Parts of a Million.	&c.

From whence it is evident, that as whole numbers *increase* by a tenfold proportion towards

wards the left hand; so decimal parts *decrease* in the same proportion towards the right.

EXAMPLE. Write down or express, fifteen parts of a hundred, in decimals.

Answer .15

Again, Write down fifty two integers, and seventy five parts of one thousand the integer.

Answer 52.075

N.B. There are finite decimals, and those which are infinite. A finite decimal is that which ends at a certain number of places; but an infinite, that which no where ends, and these are called circulating decimals or repetends, and distinguished by a dash upon the figure, and are such wherein one or more figures are continually repeated; as if 30 was to be divided by 9, the quotient would be $3.\acute{3}$; the decimal being a continual circulation of 3's, equal to the fraction of $\frac{3}{9}$. Thus $24.3\acute{6}66$, &c. is call'd a single circulate, or repetend of 6's; and $6.\acute{2}6\acute{4}$, &c. is call'd a compound circulating decimal, and has the first and last of the recurring digits dashed accordingly. Otherwise any of these expressions may be continued at pleasure, by repeating the circulating figure or figures without the dash.

In all operations, if the result consists of several nines in the decimal place, reject them, and make the next superior place an unit more; thus for 6.23999, &c. write 6.24.

ADDITION *and* SUBTRACTION.

CASE I.

To add or ſubtract finite decimals.

RULE.

HAVING ſet down all the propos'd numbers in their reſpective places (as in addition, &c. of whole numbers) viz. every figure as well of the decimal parts as of the whole number directly underneatth thoſe of the ſame value or name, which may be very eaſily done if the ſeparating points are placed directly under one another. Then, add or ſubſtract them as if they were all whole numbers, and for their ſum or difference, cut off by the ſeparating point, ſo many places of decimal parts as there are in any of the given numbers.

EXAMPLES *in* ADDITION.

102.73	19.3745	8219.801
45.0264	5.02	0.54
3.141	196.7	59.32783
563.0	3278.92303	.456
8.535	36.14	424.535207
722.4324	3536.15753	8704.660037

EXAMPLES

EXAMPLES *in* SUBTRACTION.

From	192.6521364	7163760.15
Take	78.461538	3901842.08924
	114.1905984	3261918.06076

CASE II.

To add or subtract decimals with single circulates or repetends.

RULE.

Make every line to end at the same place by filling up the vacancies with the recurring digits, and to the finite terms you may annex a cypher or cyphers, then add as before; only increase the sum of the right hand row with as many units as it contains nines, and the figure in the sum under that place will be a circulate.

And for subtraction, fill up and continue as in the following examples; and if the right hand of the subtrahend be greater than the upper figure in the minuend, instead of borrowing 10 as usual, borrow 9 in this place, the rest as usual, and the right hand figure of the remainder will be a circulate.

EXAMPLES.

Add	Subtract.	
471.6413		
15.2	8.1476	412.32
3217.846	3.9740	16.6497
3.127		

In theſe examples you obſerve there are ſingle recurring figures, which before you add or ſubtract, muſt be made to end together.

$471.641\dot{8}$		
15.2000	See the ſame filled up.	
3217.8460	$8.147\dot{6}$	$412.3\dot{2}22$
$3.12\dot{7}7$	3.9740	$16.649\dot{7}$
$3707.815\dot{1}$	$4.173\dot{6}$	$395.672\dot{4}$

24.7	$7.64\dot{8}33$	Here it may be obſerved, that in each Example the circulates are carried one place farther than the finite expreſſions.
$12.\dot{2}2$	2.7646	
$12.4\dot{7}$	$4.8787\dot{8}$	

MULTIPLICATION.

CASE I.

When both the factors are finite decimals.

RULE.

WHETHER the factors, viz. the number propos'd to be multiplied together, are either all decimal parts, or decimals join'd to whole numbers, multiply them together as if they were all whole numbers; only obſerve to point off ſo many decimal places on the right hand of the product as are in both the multiplicand and multiplier.

But when it ſo happens that there are not ſo many figures in the product as there are decimal places requir'd, in that caſe you muſt ſupply

ply the defect by prefixing cyphers to the left hand of the product.

EXAMPLES.

Multiply .45 into itself:	And 1.7345 by .69.
.45	.69
225	156105
180	104070
.2025	1.196805

Multiply 560.138 by 3.087:

```
       3.087
    --------
     3920966
    4481104
  1680414
  ----------
  1729.146006
```

And .01472 by .1045.

```
      .1045
     ------
       7360
      5888
    1472
   ----------
   .001538240
```

CASE

CASE II.

If the right hand figure of the multiplicand is a circulate.

RULE.

Multiply the multiplicand as before, by every figure in the multiplier; observing to increase the right hand figure of each resulting line, by as many units as there are nines in the first product of that line; and the right hand figure of each line will be a circulate; therefore in adding them together, make them end at the same place, as shewn in addition.

EXAMPLES.

$6174.4\dot{8}$	$5167.924\dot{8}$
86.2	156
$123488\dot{6}$	$31007546\dot{0}$
$3704660\dot{0}0$	$25839621\dot{6}6$
$4939546\dot{6}66$	$51679248\dot{}33$
$532236.15\dot{8}$	$806196.196\dot{0}$

CASE III.

If the multiplier has a single repetend.

RULE.

Multiply by it as though it was a finite digit, setting the product of this first line one place forwarder than ordinary, towards the left

left hand; divide the result by nine, continuing the quotient (if needful) till it arrives at a circulate; the rest of the work as usual; then beginning at the place under the right hand figure of the multiplicand or multiplier, cut off for fractions, and this will be the true product.

EXAMPLES.

```
       518.275        4260.95217
          .646              1.48
      ________          ________
     9)3109650      9)1278285651
      ________      ____________
       3455166       1420317390
      2073100       1704380868
     3109650        426095217
      ________      ____________
    335.151166     6107.3647770
      ________      ____________
```

CASE IV.

When both multiplicand and multiplier have each a single circulate.

RULE.

In multiplying the multiplicand by each figure in the multiplier, observe Case II. but the first line (or that line produced by multiplying the multiplicand by the repetend in the multiplier) must be order'd by Case III.

EXAMPLES.

EXAMPLES.

62.5128	48.754
542.4	72.13
9)2500515	9)146263
277835061, &c.	1625148 1
125025777, &c.	4875444 4
250051555, &c.	9750888 8
3125644444, &c.	3412811111 1
33919.769283, &c.	3516.820598

Note. From the laſt example it is eaſy to conceive how compound circulates are generated; and if it was carried on continually, it wou'd ſtill repeat the ſame.

EXAMPLES, having a compound repetend in the multiplicand.

6.84326 3	547.4624 7
4.7	32
4790284 2	10949249 4
2737305 3 0	16423874 2 2
32.163317	17518.7991

Here you multiply as uſual, adding ſo many units to the right hand place of the product made by each digit, as there are tens in the product made by the ſaid digit, and the left hand

hand leading digit of the repetend; each of which products contains a repetend equal in number of places to that in the multiplicand, which being made conterminous, and added together, gives a repetend alſo, of the like number of places in the general product.

EXAMPLE of a compound repetend in the multiplier.

Let 23.164 be multiplied by $4.\dot{2}\dot{5}$.

$$
\begin{array}{r}
23.164 \\
.\dot{2}\dot{5} \\
\hline
115820 \\
46328 \\
\hline
\end{array}
$$

$.99)5.79100(5.84\dot{4}\dot{9}$ product of repetends.

$$
\begin{array}{l}
495 \\
\hline
841 \\
792 \\
\hline
490 \\
396 \\
\hline
940 \\
891 \\
\hline
49
\end{array}
$$

$$
\begin{array}{r}
23.164 \\
4 \\
\hline
92.656 \\
5.84\dot{9}\dot{4}\dot{9} \\
\hline
\text{True product } 98.505\dot{4}\dot{9}
\end{array}
$$

Thus

Thus after multiplying by each digit of repetends as in the common way, you divide that product by as many 9's as there are repetends in the multiplier, for the product of repetends, which added to the terminate product, gives the true product. Thus you see the 9's give a denominator; and the value of the repetend is equal to this fraction $\frac{25}{99}$.

An EXAMPLE having a compound repetend both in the multiplicand and multiplier, viz. 111.94 multiply'd by 4.635.

111.94
4.600
―――――
67165
447767
―――――
514.93393
3.9575
―――――
518.89043, &c.
Product.

111.94
.035
―――――
55970
335825
―――――
.99)3.91796(3.9575
297
―――
947
891
―――
569
495
―――
746
693
―――
537
495
―――
42

N.B. When any number is to be multiplied by a repetend, in such case, multiply by the repetend as if it was a terminate number, and divide by the denominator of the repetend, viz. 9, or 99, or 999, or 9999, &c.

DIVISION.

DIVISION.

CASE I.

When the divisor and dividend are finite decimals.

DIVIDE as in division of whole numbers; and to value the quotient, observe either of the following rules.

RULE I.

The first figure in the quotient must be of the same denomination with that figure in the dividend which stands (or is suppos'd to stand) over the units place in the divisor, at first seeking.

II. The quotient must always have so many decimal places, as the dividend has more than the divisor.

Note 1st. If the divisor and dividend have both the same number of decimal places, the quotient will be a whole number.

2d. If the dividend hath not so many places of decimals as the divisor, there must be a competent number of cyphers annex'd to the dividend to make it consist of as many (at least) or more places of decimals than the divisor; for the cyphers added must be reckon'd as decimals.

3d. But if when the division is ended, the quotient has not so many places of figures as it shou'd have places of decimals, then that deficiency must be supply'd with cyphers prefixt to the left hand of the quotient, with a point before them.

4th. In dividing of whole or mixt numbers, if there be a remainder, you may bring down more cyphers, and by continuing your divifion, carry the quotient to as many places of decimals as you pleafe.

EXAMPLES.

Let 8.142 be divided by 62: And 62 by .742.

```
62)8.142(.131        .742)62.0000(83.5
   62                     5936
   ---                    ----
   194                     2640
   186                     2226
   ---                     ----
     82                     4140
     62                     3710
     --                     ----
     20                      430
```

Divide 746.54 by 3.07: And .072 by 9.05.

```
3.07)746.54(243      9.05).0720000(.00795
     614                   6335
     ---                   ----
     1325                   8650
     1228                   8145
     ----                   ----
      974                    5050
      921                    4525
      ---                    ----
       53                     525
```

CASE

CASE II.

If there be a circulate in the dividend.

RULE.

Bring down the ſaid repetend as often as you think proper, or till you come to a repetend in the quotient.

EXAMPLES.

```
8)84.36(10.54583
  8
  ——
   43
   40
   ——
    36
    32
    ——
     46
     40
     ——
      66
      64
      ——
       26 remain.
       24
       ——
       26 ſame remainder.
```

```
12)42.344(3.5286953
   36
   ——
    63
    60
    ——
     34
     24
     ——
     104
      96
      ——
       83
       72
       ——
       114
       108
       ———
         64
         60
         ——
          48
          36
          ——
           7
```

N.B. If before the quotient repeats it ſhou'd require a long or tedious operation, you may leave off at what number of places may be found proper or convenient.

CASE III.

When the divisor has a repetend.

Rule.

When any number is to be divided by a repetend, set the whole dividend under itself in such manner, that each figure may be remov'd so many places to the right hand as are the number of circulating places in the divisor; and in like manner remove the divisor, but never carry the figures in the lowermost line any further to the right hand, than the uppermost line of the divisor; then subtract as in whole numbers, and let the remainders be a new divisor and new dividend; and if there be any circulating figures in the first dividend, regard must be had to the continuance of them in subtracting one from the other, and the like number of repetends will be in this new dividend.

Examples.

```
5.3̇)74
 5  74
--------
48)66.6(13.875
   48
   ---
   186              6.4̇)6.4
   144                   64
   ---                  ----
    420             6.4)6.336(99
    384                 576
    ---                 ---
     360                 576
     336                 576
     ---                 ---
      240
      240
      ---
```

N. B.

N.B. You find it sometimes happens that the repetend begins in the whole number. And it is plain to any one's observation, that any number of digits being divided by the same number of 9's will continually circulate by the figures of the dividend, and the remainder will be the same as the said figures in the dividend. Therefore if 64 is multiplied by 100, it will exceed 64 multiplied by 99 by 64 units; the difference between 100 and 99 being unity. From whence it may be easily apprehended, that whenever any decimal fraction circulates or repeats, so many 9's will be the denominator as there are repeating places of figures in the fraction; viz. if there is a single circulate 9 is its denominator, if double 99 is its denominator, &c. for $\frac{64}{99}$ is equal to 64; and 4.5 is $4\frac{5}{9}$. Hence appears the reason of so many units being added in addition, as are numbers of 9's in the last row to the right hand, and of borrowing a 9 in subtraction, because 9 is always a denominator to these single repetends. In multiplication also, we have a reason for that method of dividing the product by 9 when the multiplier is a single repetend; or by 99 if it is a double repetend; or by 999 if it is a triple repetend, &c.

More Examples for practice.

```
41.7)274.693
41   274693
----------------
37.6)247.2240(6.575
     2256
     ----
      2162
      1880
      ----
       2824
       2632
       ----
        1920
        1880
        ----
          40
```

```
8.074)12.02472
807   1202472
----------------
7.267)10.822248(1.489
        7267
        ----
        35552
        29068
        -----
         64844
         58136
         -----
          67088
          65403
          -----
           1685
```

```
72.13)3516.820592
721   3516820592
--------------------
64.92)3165.138533(48.754
      25968
      -----
       56833
       51936
       -----
        48978
        45444
        -----
         35345
         32460
         -----
          28853
          25968
          -----
          28853
```

See the proof of this in Multiplication, Case IV.

The

The following is an anfwer to a queftion in the *Gentleman's Magazine* for Feb. 1750, which I believe was never anfwered before.

Question.

Required the true quotient of the following divifion, with 31 figures only, as perfect as if infinitely continued by the common methods, and with infinitely lefs burden to the memory?

999999)719560075657957917.(

See the work.

```
719560075657957917.0000000000000
      7195600756579579170000000
            7195600756579579170
                  7195600756579   5
                        7195600   7
                              7   1
--------------------------------
719560795218.7531357531357531357
```

Anfwer, the true quotient.

N. B. This fame method will anfwer all the purpofes of dividing any number by any quantity of 9's, by placing the dividend under itfelf in fuch a repeated order as are 9's in the divifor; and which method will be found ufeful in repetends.

REDUCTION.

REDUCTION.

CASE I.

To reduce a vulgar fraction to a decimal.

Rule.

If to the numerator given you annex a competent number of cyphers, and divide the result by the denominator, the quotient is the decimal equivalent to the vulgar fraction given.

Examples.

Reduce $\frac{1}{4}$ to a decimal: And $\frac{3}{4}$ to a decimal.

$$4)1.00 \qquad\qquad 4)3.00$$
$$\overline{.25} \qquad\qquad \overline{.75}$$

Find the decimal equal to $\frac{9}{12}$:

$$12)9.00$$
$$\overline{.75}$$

And the decimal to $\frac{10}{12}$.

$$12)10.00$$
$$\overline{.83}$$

Reduce

Reduce $\frac{3}{573}$ to a decimal: And $\frac{1286}{1378}$ to a decimal.

573)3.000000(.005235	1378)1286.0000(.9332
2865	12402
1350	4580
1146	4134
2040	4460
1719	4134
3210	3260
2865	2756
345	504

Note. If you ſhou'd have a compound vulgar fraction, and wou'd reduce it into a decimal, you muſt firſt reduce the compound fraction to a ſingle one, thus; multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, and proceed as before.

Reduce this compound vulgar fraction, viz. $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{5}$ of any thing, to a decimal faction.

5
4

20 3
2 2

40) 6.00(.15
40

200
200

And

And thus the compound vulgar fraction of 1d. is $\frac{1}{12}$ of $\frac{1}{20}$, equal to $\frac{1}{240}$ of a £.

CASE II.

To find the value of a decimal in the known parts of money, weights, measures, &c.

Rule.

Multiply the given decimal by the number of parts in the next inferior denomination, and from the product point off so many places to the right hand as there are places in the decimal given; and multiply those figures pointed off by the number of parts in the next inferior denomination, and prick off so many places as before, and so continue to do, till you have brought it to the lowest denomination requir'd.

Examples.

What's the value of .7566 of a pound sterling?

```
       20
    -------
s.  15.1320
         12
    -------
d.   1.5840
          4
    -------                s.   d.
q.   2.3360  Answer   15 : 1½ .336
    -------
```

Note. There is a more compendious method of valuing the decimal of a pound very near, only

only by the inspection of the three first figures. Double the first decimal place towards the left hand (or place of primes) and if the second figure be under 5, then the first figure only doubled will be shillings; but if the second figure be 5, or above 5, then you must add one shilling more to those you doubled; and what remains from the second figure above 5, carry to the next figure, and reckon them as so many farthings; and if they be 14 or above, abate 1; and if 28 or above, abate or cast away two farthings of the number.

What is the value of .712 of a foot? And the value of .3705 of a yard?

	.712		.3705		
	12		3		
In.	8.544		1.1115		
	4		12		
qu.	2.176	In.	1.3380	F. I.	
		Ans. 8½ .176		Ans. 1 : 1,&c.	

N.B. It may not be improper in this place to shew how to value a vulgar fraction, as it may sometimes be found more ready than reducing them into decimals.

RULE.

Multiply the integer (or quantity) into the numerator, and divide by the denominator; and if any thing remains, multiply it by the number contained in the next inferior denomination with

with respect to one of the integer, and proceed to divide as before.

What's the value of $\frac{5}{8}$ of a pound sterling?

```
        s.
        20
         5
       ———— s.  d.
    8)100(12 : 6 Facit.
        96
       ————
         4
        12
       ————
     8)48(6
        48
       ————
```

What value is 240 times $\frac{4}{15}$ of a penny?

```
         4
       ————
    ⎧ 3)960
 15 ⎨  ————
    ⎩ 5)320
       ———— s.  d.
    12)64(5 : 4 Facit.
```

And thus in like manner you may multiply by any vulgar fraction.

CASE III.

To reduce the different denominations of money weights, measures, &c. to their equivalent decimal values.

Rule.

To the number of parts of the lesser denomination given, annex a competent number of cyphers,

phers, and divide by the number of ſuch parts that are contained in the greater denomination, to which the decimal is to be brought; and the quotient is the decimal required.

EXAMPLES.

What decimal of a pound is 17 ſhillings?

2|0)17.0|0

Anſwer .85

Reduce 14s. 6d. to the decimal of a pound.

```
        s. d.
20      14:6
12      12
--      -----
24|0 ) 174.00|0(.725 Anſwer.
       168
       ---
         60
         48
         --
         120
         120
         ---
```

Let $3\frac{1}{4}$ inches be reduced to the decimal of a foot.

```
        I.
   12   3 1/4
    4   4
   --   ----------
   48 ( 13.0000(.2708 Answer.
        96
        ----
        340
        336
        -------
         400
         384
         ----
          16
```

Reduce 2qr. 25lb. to the decimal of a hundred weight.

```
                Qr. lb.
          4      2 : 25
         28     28
       -----   --------
        112 ) 81.00(.7232 Answer.
               784
               ----
               260
               224
               ----
                360
                336
                ----
                 240
                 224
                 ----
                  16
```

What decimal of a foot is 4 inches?

```
   12)4.0
      ----
Answer .3
      ----
```

N,B.

N. B. There is another rule which is somewhat shorter than the preceeding one; that is, Write the given denomination, or parts, orderly one under the other, the inferior or least parts being uppermost; let these be dividends; against each part on the left-hand, write the number contained in one of its next superior, and let these be divisors: Then beginning with the upper one, write the quotient of each division as decimal parts, on the right-hand of the dividend next below it; and let this mixt number be divided by its divisor, &c. and the last quotient will be the decimal sought.

EXAMPLES.

Reduce 8s. 4d. to its equivalent decimal of a pound sterling.

12	4
20	8.3
	.416

Thus .416 of a £. is equal to 8s. and 4d.

What decimal part of a foot is equal to 7 inches 5 parts?

12	5
12	7.416
	.61805

Answer .61805 of a foot.

What decimal of a £. is equal to 15 s. 8d. $\frac{1}{4}$?

4	1
12	8.25
20	15.6875
	.784375

Answer .784375 £.

What's the decimal of 9 oz. 16dwts. 10 gr. a lb. troy being the integer?

24	4	10
	6	(2.5
20		16.416
12		9.82083

Answer .8184027 lb. troy.

 It

It is of uſe in decimal computations, to have tables of the decimal values of the parts of coin, weights, meaſures, &c. therefore the following tables are calculated for that purpoſe.

And the manner of conſtructing theſe or any ſuch like tables, may be done either by the preceding rules, or by the following one: viz. The ſhillings 19, 18, 17, 16, &c. are ſeparately divided by 20, and the ſeveral quotients are the decimals of their reſpective ſhillings. Next the decimals of 11, 10, 9, &c. ſhillings, are divided by 12, and the quotients are the decimals of the pence 11, 10, 9, &c. Then the decimals of 3, 2, 1, pence, are divided by 4, and the quotients are the decimals of the farthings 3, 2, 1.

Alſo in the ſame manner are the other tables of weights, meaſures, &c. conſtructed, having a regard how many of a leſſer denomination are contained in a ſuperior one.

TABLE

TABLE I.

COIN, *one pound the integer.*

Shillings.	Decimals.	Pence.	Decimals.
19	.95	11	.04583
18	.9	10	.0416
17	.85	9	.0375
16	.8	8	.03
15	.75	7	.02916
14	.7	6	.025
13	.65	5	.02083
12	.6	4	.016
11	.55	3	.0125
10	.5	2	.0083
9	.45	1	.00416
8	.4	Farthings.	Decimals.
7	.35	3	.003125
6	.3	2	.002083
5	.25	1	.0010416
4	.2		
3	.15		
2	.1		
1	.05		

Note. The following table of ounces will ſerve for inches, and any thing where 12 is the integer.

TABLE II.

TROY WEIGHT, *one pound the integer.*

Ounces.	Decimals.	Dwts.	Decimals.
11	$.91\dot{6}$	10	$.041\dot{6}$
10	$.8\dot{3}$	9	.0375
9	.75	8	$.0\dot{3}$
8	$.\dot{6}$	7	$.0291\dot{6}$
7	$.58\dot{3}$	6	.025
6	.5	5	$.0208\dot{3}$
5	$.41\dot{6}$	4	$.01\dot{6}$
4	$.\dot{3}$	3	.0125
3	.25	2	$.008\dot{3}$
2	$.1\dot{6}$	1	$.0041\dot{6}$
1	$.08\dot{3}$		

Grains.	Decimals.
10	.001736
9	.001563
8	.001389
7	.001215
6	.001042
5	.000868
4	.000694
3	.000521
2	.000347
1	.000173

TABLE

TABLE III.

AVOIRDUPOIS WEIGHT.

1 Cwt. the integer.		*1 Pound the integer.*	
Pounds.	Decimals.	Ounces.	Decimals.
20	.178571	12	.75
10	.089285	10	.625
9	.080357	9	.5625
8	.071428	8	.5
7	.0625	7	.4375
6	.053571	6	.375
5	.044642	5	.3125
4	.035714	4	.25
3	.026785	3	.1875
2	.017857	2	.125
1	.008928	1	.0625
Ounces.	Decimals.	Drams.	Decimals.
10	.005580	10	.039062
9	.005022	9	.035156
8	.004464	8	.03125
7	.003906	7	.027344
6	.003348	6	.023437
5	.002709	5	.019531
4	.002232	4	.015625
3	.001674	3	.011718
2	.001116	2	.007812
1	.000558	1	.003906

Note. If you have occasion for the decimals of a ton weight, the decimals of the shillings in the first table will serve for the hundred weights, supposing one ton the integer; and for quarters, .25, .5, and .75.

TABLE

TABLE IV.

LIQUID MEASURE.

1 Hhd. the integer.

Gall.	Decimals.
60	.952381
50	.793651
40	.634921
30	.47619
20	.31746
10	.15873
9	.142857
8	.126984
7	.1
6	.095238
5	.079365
4	.063492
3	.047619
2	.031746
1	.015873
Pints.	**Decimals**
7	.013889
6	.011905
5	.009921
4	.007937
3	.005952
2	.003968
1	.001984

TABLE V.

LONG MEASURE.

1 Yard the integer.

Feet.	Decimals.
1	.6
2	3
Inch.	**Decim.**
11	.305
10	.27
9	.249
8	.2
7	.194
6	.16
5	.138
4	.1
3	.083
2	.05
1	.027
Qu. I.	**Decim.**
3	.02083
2	.0138
1	.00694

Note. When you want the decimal of a foot for any number of inches, you may make uſe of the decimal annexed to ſuch number of ounces in table II.

TABLE

TABLE VI.

Time.

An Year the integer.

Days.	Decimals.
300	.821918
200	.547945
100	.273973
90	.246575
80	.219178
70	.191781
60	.164383
50	.136986
40	.109589
30	.082192
20	.054794
10	.027397
9	.024657
8	.021918
7	.019178
6	.016438
5	.013698
4	.010959
3	.008219
2	.005479
1	.00274

N. B. In making uſe of the tables, if the number of parts wanted are not found therein, take it out at twice; thus, for 15 dwts. add 10 and 5 together; and the like for any other.

The

The uſe of the foregoing tables.

What decimal of a pound ſterling is 14s. and 10d. ½?

In Table I.

Againſt	14s.	is	.700000
Againſt	10d.	is	.041666
Againſt	0d. ½	is	.002083

These added, their ſum is .74375, the decimal [required.

What decimal part of a year is 200 days?

Anſwer .547945

What decimal of a foot is 7 inches and 3 parts?

12).25

.02083
.58333

Anſwer .60416

N.B. In making uſe of the ſecond table in this caſe, you muſt divide the decimal againſt 3 in the table by 12 for the parts, which added to the decimal of 7 for inches, gives the anſwer right.

What is the decimal of 1 foot 4 ½ inches, a yard being the integer?

.3333
.1111
.0138

.4583 Anſwer.

A short and easy method of reducing troy weight and avoirdupois weight into each other. Multiply the pounds (and decimal parts of a pound) avoirdupois by 1.21527, and it gives the like value in troy weight; and multiplying the pounds troy by .822857 $\frac{1}{7}$ gives its equivalent value in avoirdupois weight. These factots are found by dividing the number of grains in a pound by each other.

EXAMPLES.

Reduce 202 $\frac{3}{4}$ lb. avoirdupois into troy.

```
          1.21527
           202.75
        ---------
           607638
          8506944
        24305555
      2430555555
     ------------   lb.  oz. dwts. gr.
Ans. 246.3975694 or 246 : 4 : 15 : 10
```

Reduce 175 lb. troy into avoirdupois.

```
        .822857 1/7      7)175
            175           ---
        -----------        25
          4114310
         5759999
         822857
        ---------
Answer  144.000000
```

Note.

Note. The troy pound is to the avoirdupois pound in the ſame proportion as the wine gallon is to the ale gallon.

Or as there are 5760 grains contained in the pound troy, the avoirdupois pound conſiſts of 7000 grains, which two numbers are alſo proper factors to reduce theſe weights into each other; viz. if you would reduce troy pounds into avoirdupois pounds, multiply the numbers of pounds into the leſſer number and divide by the greater; obſerving the contrary for troy pounds. Otherwiſe 576 and 700; or 144 and 175 are factors which will anſwer the ſame purpoſe. But as the pound troy is leſs than the avoirdupois, yet the ounce is greater; therefore if you would reduce the ounces into each other, multiply the troy ounces by 56 and divide by 51 for avoirdupois ounces; and for troy ounces, multiply the avoirdupois ounces by 51 and divide by 56.

PROPORTION *or* The RULE OF THREE

IS when there are three numbers given to find out a fourth.

It is the rule of three Direct when the three given numbers (or terms) require a fourth number that ſhall bear the ſame proportion to the third as the ſecond does to the firſt; and here the ſecond and third terms are to be multiplied together, and their product divided by the firſt.

The

The proper method of ſtating queſtions in this rule, is thus,

As 4 : 12 : : 8 : 24.

viz. as 4 is to 12, ſo is 8 to 24.

It is called the rule of three Indirect when the 3 numbers given require a fourth that ſhall bear the ſame proportion to the ſecond as the third does to the firſt; and in this caſe the firſt and ſecond numbers are multiplied together, and their product divided by the third.

In ſtating all queſtions in proportion (whether they require a ſingle or a double ſtating) obſerve where the demand lies, and make that your third number, and ſee the firſt and third numbers are of one denomination, or ſo reduced; and your ſecond or middle number is always of the ſame name with the fourth number or anſwer required, and muſt be reduc'd into its loweſt denomination mentioned.

When a queſtion is ſtated, if the third number, being more than the firſt requires more, or, being leſs requires leſs, it is *direct*; but if the third number, being more, requires leſs, or being leſs, requires more, it is *indirect*.

N. B. The queſtions I have made choice of are promiſcuouſly inſerted, in order to ſtrengthen the judgment of the young practitioner; and which method of practice I apprehend to be the beſt.

EXAMPLES.

If I give 2 guineas for $3\frac{1}{2}$ yards, what quantity ſhall I have for 21l. 10s. at the ſame rate?

```
       £.    yds.       £.
  As 2.1 : 3.5. : : 21.5
                     3.5
                   -----
                    1075
                    645
               ---------  Yds.
           2,1)75.25 (35.83 Anſwer
               63
               ----
                122
                105
                -----
                 175
                 168
                 -----
                   70
                   63
                   ---
                    7
```

If my yearly income be 39 l. what is that per week?

```
       w.   £.    w.
  As 52 : 39 : : 1
          20     s.
      ------ (  15
    52)780   (  Anſwer.
       52
       ---
       260
       260
       ---
```

If a rule of 2 foot long give 3 ½ feet in ſhade, how high is that ſteeple which gives 43 yards in ſhade?

```
                         43
                          3
                         ---
As 3.5 : 2 : :          129
                          2
                     -------- F.
                 3.5)258.0(73.7 Anſwer.
                     245
                     ----
                      130
                      105
                      ----
                       250
                       245
                       ---
                         5
```

If 42lb. coſt 17s. 6d., what is that per lb?

```
As 42 : 17.5 : : 1
      42)17.5 (.416
         168  (  12
         ---    ---
          70  5.000
          42
          --        Anſwer 5d.
          280
          252
          ---
           28
```

How many yards of matting of ½ yard wide, will be ſufficient to cover a floor that is 16 feet wide, and 28 feet long?

```
As 16  :  28  : :  1.5
          16
        ----
         168
          28
      ---------
  1.5)448.0  ( 3)
       30      298.6
      ----     -----
       148     99.5 yds.
       135      Anſwer.
      ----
        130
        120
       ----
         100
          90
         ----
          10
```

What is the intereſt of 320 l. at 4 ½ per cent. per annum?

```
As 100 : 4.5 : : 320
                 4.5
               ------
                1600
               1280
              -------
        1|00)14|40.0
                20
              ------
                8.00
```

Anſwer 14l. 8s.

If 112 lb. weight at London make 99 lb. at Lisbon, how many lb. at London are equal to 1049 lb. at Lisbon?

As 99 : 112 :: 1049

```
                  1049
                   112
                ------
                  2098
                 1049
                1049
          ------------
          99)117488( lb.
              99      1186.74
              ---     Anſwer.
              184
               99
              ---
              858
              792
              ---
               668
               594
               ---
                740
                693
                ---
                 470
                 396
                 ---
                  74
```

If 75l. is put to intereſt, and at the end of 12 months it amounts to 81l. at what rate per cent. was the intereſt allow'd?

```
81          As 75 : 6 :: 100
75                          6
--                        ---
 6                   75)600(8l. Anſwer.
                        600
```

An acre of land contains 40 perches in length, and 4 in breadth; what muſt be the length to make an acre when the breadth is but 15 yards?

```
 5.5         As 22 : 220 : : 15
   4                  22
 ----                ----
 22.0                 440
                     440
     5.5            -----    Yards     yds.  f.
      40         15)4840   ( 322.6 or 322   2
     ----            45        Anſwer.
    220.0           -----
                     34
                     30
                     --
                     40
                     30
                     ---
                     100
                      90
                      ---
                      10
```

If a man can do a piece of work in 6 days when the day is 8 hours long; in what time will the ſame be done when the day is 10 hours long?

```
As 8 : 6 : : 10
       6
    ----
 10)48
 ------
 d.  4.8
      24
 ------
 h, 19.2
      60
 ------                 D.  H.  M.
 m. 12.0   Anſwer       4   19  12
```

If when the bushel of wheat costs 3s. 4d. the penny loaf weighs 12 ounces; what will be the weight of a 9 penny loaf (at the same rate) when the bushel costs 4s. 4d?

```
     s. d.  oz.    s. d.
  As 3  4 : 12 ::  4  4
     12            12
    ----          ----       d.    oz.     d.
     40            52     As 1 : 9.23 :: 9
     12                              9
   ------                      --------
52)480(9.23                   16)83.07(5.19
   468                           80     16
   ----                          ---   ----
    120   lb. oz.                 30    114
    104   5   3                   16    19
    ---   Answer.                ---   ----
     160                         147   3.04
     156                         144
     ---                         ---
       4                           3
```

In 145l. 12s. sterling, how much flemish money at the rate of 1l. 13s. 4d. to the pound sterling?

```
         £.   s. d.     £.
     As  1 :  33 4 :: 145.6
              12         400
             ----   ----------
              400  12)58240.0
                 -------------
                  2|0)485|3      4
                     -------------
    Answer £. 242 13 4
```

Bought

Bought a quantity of braſs at 4l. 6s. per C. how muſt I ſell it to gain 5 per cent.?

As 100 : 4.3 :: 105
4.3

315
420

1|00)4|51.5
20

10.300
12

3.6 . .
4

2.4

£. s. d.
4 10 3 ½ per C.
Anſwer.

Suppoſe 75 gallons of water do in one hour fall into a ciſtern containing 250 gallons, and by a pipe in the ſame ciſtern there runs out at the ſame time 51 gallons in the hour; in how many hours will it be filled?

75
51

24

As 24 : 1 :: 250
24) 250 (10.416
24 60

100 25.000
96

40
24

160
144

16

H. M.
Anſwer 10 25

Suppose I deliver to a refiner a quantity of waste metal, averdupois weight, in order to have it refined, and he agrees to return 95lb. of 100 troy weight; what factor shall be found to multiply the given weight averdupois into the return'd pounds troy, at the rate of allowance for waste, &c. of 5 lb. in the 100 troy?

N. B. I have inserted in page (35) the proper factors for reducing avoirdupois weight and troy weight into each other. Now as 1.21527 is the factor for reducing avoirdupois pounds into troy, the number required is found by this proportion; As 1 integer is to 1.21527, so is the .95 decimal of an integer to the factor sought for: See the work.

As 1 : 1.21527 : : .95
.95

607638
1093750

Answer 1.1545138

A borrow'd of B 40 l. for 6 months; afterwards B borrow'd of A 60 l. how long ought he to keep it?

As 40 : 6 : : 60
6

6|0)24|0

Answer 4 months.

If

If by laying out a ſum of money 1s. brings 14d $\frac{3}{4}$, what is gained per cent. ?

```
     s.  d.       £.
As 1 : 2 3/4 : : 100
                  20
               -----
                2000
                2.75
           ---------
          12)5500.00
           ---------
          2|0)45|8 (4
           ---------
             £.  s. d.
   Anſwer  22  18  4
```

Bought a quantity of old ſilver for 8l. 1s. which weighed 3 $\frac{1}{2}$ lb. troy; what did the ſame lie me in per ounce ?

```
   lb.oz.  £. s.      oz.
As 3 6  :  8  1  : :  1
   12      20
   --      ----
   42)     161  ( 3.83
           126      12
           ---    -----
            350  10.00
            336
            ---
             140
             126
             ---
              14
                s.  d.
        Anſwer  3   10
```

If 100 l. principal gain 5 l. intereſt in 12 months, what principal will gain as much in 5 months ?

```
     M.    £.    M.
As  12 : 100 : : 5
          12
         ----
       5)1200
         ----
Anſwer 240 £.
```

Suppoſe

Suppoſe I ſee a flaſh of lightning *, and count 6 ſeconds before I hear the thunder, how far is the thunder cloud from me ?

N. B. It is well known both by theory and experiments, that ſound flies at the rate of 1142 feet per ſecond.

	S.	F.	S.
	As 1 :	1142 : :	6
		6	
			Mile yds.
1760 yds. in a mile.		6852	feet or 1 and 524
3		5280	Anſwer.
5280		3)1572	
		524	

---

* From Mr. Franklin's experiments, it appears, that Lightning is only electrical fire drawn off from the clouds; and this electrical fire has in reality been collected from the clouds during thunder, by means of iron bars or tin tubes, in many parts of Europe, but more particularly in America where Mr. Franklin made his experiments: Therefore it were much to be wiſhed that Mr. Franklin's bars were more generally uſed than they are, for the preſervation not only of public and private buildings, but alſo for the preſervation of the lives of men and beaſts; as Mr. Franklin has made appear in his experiments; to which work I refer the reader, for a more particular account.

PRACTICE,

PRACTICE,

IN general, is a ſhort way of finding the value of any quantity by the given price of one integer; but to give all the diverſity of methods that are applicable to the rules of practice, wou'd be an endleſs task; therefore it may ſuffice in this place to take notice of the moſt general of thoſe rules; which is, to divide the quantity by ſuch aliquot part or parts as is the given price, &c. Alſo the ſame methods muſt be taken in working of decimal parts as in whole numbers.

A TABLE OF ALIQUOT PARTS.

Parts of a Pound. s. d.		Parts of a Shilling.		Parts of C. wt.	
10 0	is $\frac{1}{2}$	6d.	is $\frac{1}{2}$		
6 8	is $\frac{1}{3}$	4d.	is $\frac{1}{3}$	56lb.	$\frac{1}{2}$
5 0	is $\frac{1}{4}$	3d.	is $\frac{1}{4}$	28	$\frac{1}{4}$
4 0	is $\frac{1}{5}$	2d.	is $\frac{1}{6}$	16	$\frac{1}{7}$
3 4	is $\frac{1}{6}$	$1\frac{1}{2}$d.	is $\frac{1}{8}$	14	$\frac{1}{8}$
2 6	is $\frac{1}{8}$	1d.	is $\frac{1}{12}$	8	$\frac{1}{14}$
2 0	is $\frac{1}{10}$	$\frac{3}{4}$d.	is $\frac{1}{16}$	7	$\frac{1}{16}$

EXAMPLES.

At 3s. 3d. per lb. what will 58lb. come to?

2s.	$\frac{1}{10}$	58	2s.	$\frac{1}{10}$	58
1s.	$\frac{1}{2}$	5 16	1s.	$\frac{1}{2}$	5.8
3d.	$\frac{1}{4}$	2 18	3d.	$\frac{1}{4}$	2.9
		0 14 6			0.725
Anſwer		£ 9 8 6	or thus		£ 9.425 decimally.

At

At 5 s. 4 d. ½ per yard, what will 222 yards come to?

4s.	$\frac{1}{5}$	222
1s.	$\frac{1}{4}$	44.4
4d.	$\frac{1}{3}$	11.1
½d.	$\frac{1}{8}$	3.7
		.4625

			£.	s.	d.
Anſwer	59.6625	or	59	13	3

At 5 d. ½ per oz. what will 360 ¾ ounces come to?

4d.	$\frac{1}{3}$	360.75
1½d.	$\frac{1}{8}$	120.25
		45.09375

20)165.34375

			£.	s.	d.
Anſwer	8.267, &c.	or	8	5	4

At 1s. 9d. per ell, what will 387 yards come to at that rate?

d.		
6	½	309.6
3	½	154.8
		77.4

2|0)54|1.8

£. 27 1 9½.4
Anſwer.

4
5)1548
309$\frac{3}{5}$

5)3.0
.6

.8
12
d. 9.6
4
f. 2.4

At

At 4l. 13s. 6d. per C. wt. what will 425 C. 3 Q. 9 lb. come to?

		£.	s.	d.				
		0	10	0	$\frac{1}{2}$	425		
						4		
		4	0	0		1700		
		0	2	0	$\frac{1}{5}$	212	10	
		0	1	0	$\frac{1}{2}$	42	10	
		0	0	6	$\frac{1}{2}$	21	5	
						10	12	6
2qr	$\frac{1}{2}$	4	13	6				
1qr.	$\frac{1}{2}$	-	-	-	- - -	2	6	9
7lb.	$\frac{1}{4}$	-	-	-	- - -	1	3	4 $\frac{1}{2}$
1lb.	$\frac{1}{7}$	-	-	-	- - -	0	5	10—.5
1lb.		-	-	-	- - -		0	10—.07, &c.
								10—.07, &c.
						£.	s.	d.
		Answer				1990	15	1 $\frac{1}{2}$.64, &c.

FELLOWSHIP or COMPANY.

WIthout taking any notice of the common way of working fellowſhip, obſerve this more compendious rule: Divide the whole gain or loſs, by the whole ſtock, and multiply the quotient by each perſon's particular ſtock; the ſeveral products are the reſpective gains of each.

By this rule, bankrupts eſtates, legacies, &c. are adjuſted, in caſe of a deficiency in effects, &c.

Note.

Note. If there be any ſhillings, &c. in any one's ſtock, you muſt firſt reduce it to the decimal of a pound, and proceed on as aforeſaid. And in queſtions with time, you muſt multiply each particular ſtock by its reſpective time, and add all the products together; and in the rule, call it your whole ſtock, &c.

Examples.

A, B, and C, trading together; A, puts into ſtock 500l. B, puts 700l. and C, puts in 1200l. and the whole gain is 836l; what ſhare of it belongs to each?

	£.	
	500	
	700	
	1200	
	2400	
24|00)836|.00 (.3483	.3483	.3483
72	500	1200
116	174.16	6966
96	A's ſhare.	34833
200		418.00
192		C's ſhare.
80	.3483	
72	700	
8	243.83	
	B's ſhare.	

	£.
A.	174.16
B.	243.83
C.	418.00
Proof	836.00

A legacy being left as follows, *viz.* to A. the value of 500l. between B and C 350l, and to D 100l.; but by loſſes before the dividends could be paid off, the legacy ſuffer'd 250l. what does each dividend come to bearing a part proportionable in the loſs?

```
500
350     950)700.000(.73684      .73684          .73684
100         6650                   500             350
———         ————————            ————————        ————————
950          3500              368.42000        3684200
250          2850              A.       20     221052
———         ——————                 ——————      ————————
700          6500                   8.4        257.89400
             5700                    12        B.&C.   20
            ——————                 ——————          ————————
             8000                   4.8           17.88
             7600                     4              12
            ——————                 ——————          ————————
              4000                  3.2           10.56
              3800                                    4
             ——————                                ————————
               200                                   2.24
```

Answer.	£.	s.	d.
A's ſhare	368	8	$4\frac{3}{4}$
B & C's ſhare	257	17	$10\frac{1}{2}$
D's ſhare	73	13	8
			$\frac{3}{4}$ rem.
Proof	700	0	0

```
 .73684
    100
————————
73.68400
D.    20
————————
  13.68
     12
————————
   8.16
```

A put

A put into company 525l. 10s. for 6 months, B, 382l. 15s. for 8 months, and C, 1000l. for 4 months; they gain in all 1286l. 12s.; how much is that for each?

```
525.5      382.75      1000
    6           8         4
------     -------     ----
3153.0     3062.00     4000
3062
4000
-----
10215)1286.600000(.125952
       10215
       -----
        26510
        20430
        -----
         60800
         51075
         -----
          97250
          91935
          -----
           53150
           51075
           -----
            20750
            20430
            -----
              320
```

```
      .125952
         3153
   ----------
       377856
      629760
     125952
    377856
   ----------
A.  397.126656
```

```
      .125952
         3062
   ----------
       251904
      755712
    3778560
   ----------
B.  385.665024
```

```
      .125952
         4000
   ----------
C. 503.808000
```

A's share	397.126656
B's share	385.665024
C's share	503.808
Total	1286.599680
Or	1286l. 12s. very near.

Suppose

Suppofe a bankrupt is indebted 2050l. 10s. and his effects amounts but to 715l. 5s. what is that in the pound?

```
2050.5)715.25(.3488          .3488
        61515                   20
        ------                ------
        100100                6.9760
         82020                    12
        ------                -------
        180800               11.7120
        164040                     4
        ------                ------
         167600               2.8480
         164040
         ------          Anfwer 6s. 11d. 1/2
           3560
```

Three merchants A B and C freight a fhip with 96 ton of wine: Thus A loaded 24 ton, B 32, and C 40 ton; but by extremity of weather they caft 12 ton overboard; how much muft each merchant bear of this lofs?

```
 24            96)12.000(.125           .125
 32                96                     24
 40               ---                   ----
---               240                    500
 96               192                    250
                  ---                   -----
        .125      480           Ton     3.000
          32      480                A's lofs.
        ----      ---   .125
         250              40
         375            ----
        -----
B's lofs 4.000          5.000
                      C's lofs.
```

Suppofe

Suppose 20 shillings to be shared among 3 persons; A, to have $\frac{1}{2}$; B, $\frac{1}{3}$; and C, $\frac{1}{4}$: what is the share of each in proportion one with another?

s.	s.	s.
$\frac{1}{2}$)20	$\frac{1}{3}$)20	$\frac{1}{4}$)20
10	6.6	5.0
	6	5
	6.0	4.5

.5
.3
.25
———
1.083
108
———
.975) 10 (s. 10 10 9.000 (s. .230769 A's share.

```
           s.
           10
           10
.975)     9.000 ( .230769
          8775  ( A's share.
          ----
          2250
          1950
          ----
          3000
          2925
          ----
           7500
           6825
           ----
            6750
            5850
            ----
             9000
             8775
             ----
              225
```

```
      s.           s.
.975)6.000 ( 6.153846
     5850  ( B's share.
     ----
     1500
      975
     ----
     5250
     4875
     ----
      3750
      2925
      ----
       8250
       7800
       ----
        4500
        3900
        ----
         6000
         5850
         ----
          150
```

See the remainder of the work in the next page.

```
.975)4.500(4.615384        225 }
     3900  C's share.      150 } remainders
     ----                  600 }
      6000                 ---
      5850                 975
      ----                 ---
       1500
        975
       ----                     s.
       5250                 9.230769
       4875                 6.153846
       ----                 4.615384
        3750                 .000001 for the re-
        2925                ---------  mainder.
        ----          Proof 20.000000
         8250
         7800
         ----
          4500
          3900
          ----
           600
```

Three merchants being in partnership 16 months: A put into common stock 100l. and at 8 months end took out 40l. and at 4 months after put in 140l. B put in at first 200l. and at 6 months end put in 50l. more, and at 4 months after took out 150l. C put in at first 150l. and at 4 months end took out 50l. and at 8 months after put in 100l. and they gained 357l. What must each person have proportionable to his time and stock?

See the work in the next page.

100	200	150
8	6	4
800	1200	600
60	250	100
4	4	8
240	1000	800
200	100	200
4	6	4
800	600	800
240	1000	800
800	1200	600
1840	2800	2200

1840
2800
2200

6840 total sum of the products.

684|0)357.00000|0(.052192
3420
1500
1368
1320
684
6360
6156
2040
1368
672

.052192	.052192	.052192
1840	2800	2200
2087680	41753600	10438400
417536	104384	104384
52192	B. 146.137600	C. 114.822400
A. 96.033280		

£.	
96.03328	A's share.
146.1376	B's share.
114.8224	C's share.
356.99328	total or very near.

SIMPLE INTEREST.

INterest in general may be work'd by the rule of three; but there are speedier ways to perform the same by proper factors, and other customary methods.

The annual interest of any sum of money is found by only multiplying the given principal by the interest of one pound for a year, which at 4 per cent. the ratio is .04; at 4 ½ per cent. the rate is .045; at 5 per cent. the ratio or interest is .05, &c. found by dividing the rate of interest by 100. Thus you may find ratios at what rate per cent. you please.

Examples.

What's a year's interest of 536 l. at 5 per cent.?

536
.05

		£.	s.
Answer	26.80 or	26	16

Thus a year's interest of any sum being found, 'tis only multiplying that sum by 2, 3, 4, &c. and the product is the interest of the said sum for 2, 3, 4, &c. years.

What's the interest of 623 l. for 3 years, at 4 per cent. per annum?

623
.04

24.92
3

		£.	s.	d.
Answer	74.76 or	74	15	2 ¼

Again,

Again, if the time required be months, 'tis only dividing the year's amount by ſuch aliquot parts according to the rules of practice as the given months are of an year.

What's the intereſt of 539 l. for 4 months, at $4\frac{1}{2}$ per cent. per annum?

		539
		.045
		2695
		2156
M.		
4	$\frac{1}{3}$	24.255
		8.085 Anſwer £. 8 s. 1 d. $8\frac{1}{4}$

What will the intereſt of 729l. 10s. 6d. come to in a year and 5 months, at 5 per cent. per annum?

12	6				729.525
20	10.5				.05
		M.			
	.525	3	$\frac{1}{4}$		36.47625
		2	$\frac{1}{6}$		9.11906
					6.07937
			Anſwer		51.67468 or £. 51 s. 13 d. $5\frac{3}{4}$

But if it be required to find the intereſt of a ſum of money for any number of days, you may either multiply the year's amount by the number of days propoſed, and divide the product by 365, &c. or (which will be ſometimes ſhorter) divide the year's amount by 365, and multiply the quotient by the number of days propoſed.

What

What intereſt will 563 l. amount to in 126 days, at 5 per cent. per annum?

```
    £.                   126
    563                 .077
    .05                -----
   -----                 882
365)28.15(.077          882
    2555                -----
   -----               9.702    £. s. d.
    2600               -----
    2555               Anſwer   9 14 0 ½
   -----               --------------------
     45
```

N. B. There is another ſhort and cuſtomary method of working ſimple intereſt: 'Tis done by multiplying the principal or given ſum by 2, 3, 4, or 5, &c. according to the rate per cent. and cut off two figures towards the right hand, which is the ſame as multiplying by the other ratios, (or dividing by 100;) then multiply the figures cut off to the right hand by 20, and take in the odd ſhillings, and cut off two figures as before; the like muſt be done by multiplying the remainder by 12 for pence, taking in the odd pence, and cutting off as before, &c.

EXAMPLES.

What is the intereſt of 275l. 10s. for a year, at 5 per cent.?

```
                        5
            -------------
    £.      13|77   10
                20
            ---------
    s.      15|50
                12
            ---------          £. s. d.
    d.       6|00    Anſ.   13 15 6
```

Note.

Note. By this method, merchants and traders estimate their allowances upon some sort of goods, which is called the discount on such goods. For example, suppose I buy 20 dozen of buckles at 5s. per doz. with an allowance of 10 per cent. discount from the maker; and I buy 20 doz. of the same of another maker at 5s. and 6d. per doz. with the discount of 20 per cent. which do I buy cheapest? The answer produces 2s. gained in the 20 dozen at 5s. and 6d.

When the given rate per cent. is not even pounds, but is $3\frac{1}{2}$, $4\frac{3}{4}$, or the like, then multiply the principal or given sum by the even pounds, and take the $\frac{1}{2}$, or $\frac{1}{4}$, or $\frac{3}{4}$ of the said principal or given sum and add to that product; then cut off two figures, and proceed as before.

What comes the interest of 585l. 2s. 6d. to, at $4\frac{1}{2}$ per cent.?

		585	2	6
				4
s. 10	$\frac{1}{2}$	2340	10	0
		292	11	3
£.		26|33	1	3
		20		
s.		6|61		
		12		
d.		7|35		
		4		
q.		1|40		

	£.	s.	d.
Ans.	26	6	$7\frac{1}{4}$

Lent 410l. upon a mortgage, to receive interest at $4\frac{3}{4}$ per cent. till it was paid off; and the mortgage being paid off 3 years 8 months after; what was the interest upon the same?

s.		£.	s.
		410	
		4	
10	$\frac{1}{2}$	1640	
5	$\frac{1}{2}$	205	
		102	10
	£.	19|47	
		20	
		9|50	
		12	
		6|00	

M.		£.	s.	d.
		19	9	6
				3
6	$\frac{1}{2}$	58	8	6
2	$\frac{1}{3}$	9	14	9
		3	4	11
	£.	71	8	2
		Answer.		

Here you also see the manner of taking in the even parts of a year; but if the interest required is for such a number of days as cannot well be taken in their even parts, first find the interest of one year as before; and for the days, you may work them by proportion; *viz.* As 365 days is to the year's amount, so is so many days to the answer requir'd.

Suppose

Suppose the interest of one year is 12l. 5s. 6d. what will be the interest of 69 days at that rate?

```
    D.     £. s. d.    D.
As 365 : 12  5  6 :: 69              12| 6
         20                          20| 5.5
        ---                            -----
        245                             .275
         12
        ----
        2946
          69
        -----
       26514
      17676
   -----------      12)
   365)203274       (556    ¾
       1825       ----------
       -----      2|0)4|6  4
        2077         19
        1825      ----------
       -----     £ 2  6  4¾
        2524     Answer.
        2190
       -----
         334
           4
       -----
   365)1336 (3
       1095
       -----
        241
```

Or thus decimally.

```
As 365 : 12.275 :: 69
                   69
             -------
             110475
             73650
             --------       £.
        365)846.975(2.320
            730          Ans.
            -----
            1169
            1095
            -----
             747
             730
            -----
             175
```

But if we suppose the principal to be 1, the rate 1, and the time 1, we can easily construct a small yet universal table for any principal, any number of days, or for any rate of interest: Thus if the principal, time and rate are 1 each,

these

these multiplied together produce 1; which divided by 365 gives .002739726, which gives the first number in the following table; and if it is multiplied by 2, 3, 4, 5, &c. it will give the other numbers. Hence it is pla[illegible] from the construction of the table, that if any principal, time and rate is multiplied together, and the product taken out of the table, that will be the interest required for the given sum, time, and rate of interest.

TABLE.

1	2739726
2	5479452
3	8219178
4	10958904
5	13698630
6	16438356
7	19178082
8	21917808
9	24657534
10	27397260

Note. If the product hath

figures,	point off decimals
7	5
6	6
5	7
4	8
3	9
2	10
1	11

Examples.

What is the interest of 110l. for 8 days at 5 per cent.?

110	4000 =	.10958904
40 *	400 =	.010958904
4400	Answer	.120547944 or 2s. 4d ¾

* Here I multiply by 40, which is the same thing as if I multiplied by 8 and by 5.

What

What is the intereſt of 160l. for 144 days at 5 per cent.?

$$\begin{array}{r} 144 \\ 160 \\ \hline 8640 \\ 144 \\ \hline 23040 \\ 5 \\ \hline 115200 \end{array}$$

$$\begin{array}{r} 100000 = 2.739726 \\ 10000 = .2739726 \\ 5000 = .13698630 \\ 200 = .005479452 \\ \hline 115200 = 3.156164352 \\ \hline \end{array} \quad \text{Anſ.}$$

But notwithſtanding what has been ſaid on ſimple intereſt for days or any part leſs than an year, the true intereſt ſhould be reckoned according to ſuch progreſſive order as compound intereſt for ſuch time; the true time of ſimple intereſt being limited to one year only. Therefore if a man pays intereſt for 100l. half yearly at 5 per cent. he ſhould not pay 2l. 10s. as is generally ſuppoſed, but £. 2.4695, that is 2l. 9s. 4d. ½ .72. And if the intereſt be paid in leſs than half an year (as aforeſaid) the error will be greater; to demonſtrate which, obſerve what follows.

It is well known that the intereſt or amount of one pound for any time greater than an year is found by raiſing the amount of one pound to the power of the time; ſo for any part of an year it is found by extracting the root of the power of the time.

A Table ſhewing the amount of 1l. from 1 month to a year.

Months	£.
1	1.004074
2	1.008169
3	1.012272
4	1.016390
5	1.020537
6	1.024695
7	1.028870
8	1.033061
9	1.037270
10	1.041496
11	1.045740
12	1.050000

For example, if you wou'd know how much a perſon ſhould pay quarterly for intereſt of 100 l. at 5 per cent, look in the table for 3 months, and againſt it you will find 1.012272, which multiplied by 100 becomes 101.2272 for the amount of 100l. in $\frac{1}{4}$ of an year, and the intereſt is 1.2272 or 1l. 4s. 6d. $\frac{1}{2}$ inſtead of £. 1.25 or 1l. 5s.; the error in this caſe being 5d. $\frac{1}{2}$ for $\frac{1}{4}$ of an year. Therefore if a man pays intereſt for money quarterly he ought to have 22d. per cent. per annum abated him; and if he pays it half yearly he ought to have 14d. $\frac{3}{4}$ or 15d. per cent. per annum abated him. Nevertheleſs an error in this caſe (its boundaries being limited within the year) is very inconſiderable to that common error of reckoning diſcount like intereſt.

REBATE

REBATE OR DISCOUNT,

ACcording to simple intereſt, is to abate in receiving money, as much as the intereſt comes to, for the time 'tis paid before it becomes due. The reaſon of this is evident from the following propoſition.

Suppoſe 100l. preſent money would diſcharge a debt of 105l due a year to come, rebate being made at 5 per cent. becauſe 100l. being put to intereſt for a year at the ſaid rate wou'd regain the 5l.

To work queſtions in this rule, obſerve this proportion; As 100l. with the intereſt for the time, is to the intereſt alone, if you wou'd find the rebate, (or to 100l. if you would find the preſent money;) ſo is the debt or ſum propounded, to the anſwer required.

EXAMPLE. I have a bill of 21 l. payable an year hence; what is the worth of it in preſent money if rebate is allowed at 5 per cent.?

As 105 : 100 : : 21
21

105)2100(20£.
210 Anſwer.

0

EXAM-

EXAMPLE. Sold goods to the value of 73l. 5s, to be paid in a year's time; what muſt be diſcounted for the preſent payment, if rebate be allowed at $4\frac{1}{2}$ per cent?

```
     £.      £.      £.
As 104.5 : 4.5 : : 73.25
                     4.5
                 -------
                   36625
                  29300
                 -------
        104.5)329.625 (3.1543
              3135          20
              ----    --------
              1612     3.0860
              1045         12
              ----    --------
               5675    1.0320
               5225
               ----
                4500
                4180
                ----
                 3200            £.  s.  d.
                 3135    Anſwer  3   3   1
                 ----
                   65
```

Now the preſent practice of diſcounting bills, &c. is done by the intereſt of the whole ſum for ſo long a time; but ſuch ſum cannot be eſteem'd a principal, nor in full value, till the time of payment is expir'd; therefore leſs intereſt muſt be requir'd according to the juſt rule of diſcount or rebate on ſuch payment, which will be the true intereſt, and anſwers to the propoſition.

EXAM-

EXAMPLE. What present money will discharge a debt of 100 l. payable at the end of 12 months, rebate being made at 5 per cent.?

```
As 105 : 100 :: 100
              100
   ------------
   105)10000 ( 95.238
        945        20
       ----    -------
        550     4.760
        525        12
       ----    ------
         250    9.120
         210
        ----
          400
          315
         ----
           850
           840
          ----
            10
```

Answer £. 95 s. 4 d. 9.

And here in this last example it will be obvious to any one's understanding, that to reckon discount like interest, is false and erroneous; for by the present practical method there would have been 5l. deducted from the 100l. instead of 4l. 15s. 3d, which is 4s. 9d. too much in the 100l.

Note. Tho' it is with interest, that after having found the interest for 12 months, the half of that sum may do for 6 months, and the 4th for 3 months, (the error in that case being but small), but in rebate it is quite otherwise.

Again,

Again, for instance, the rebate of 500l. for a year, present payment, at 5 per cent. per annum, comes to no more than 23l 16s. 2d. $\frac{1}{2}$, whereas in common practice, 25l. would be taken for the present payment of that sum due a year hence; which fallacy is very evident, for the 25l. is the interest of 500l. (present worth or principal) for a year, and is a different thing from discount. In short, by the present custom of discounting sums of money, &c. he that advances the money gets in effect the interest of what the discount falls short of interest more than the rebate comes to. Thus upon tryal you'll find it very apparent, that the discount cannot be 25l. for then the present money would be but 475l. whereas it is 476l. 3s. 9d. $\frac{1}{2}$, the interest of which for 12 months at 5 per cent. is 23l. 16s. 2d. $\frac{1}{2}$ (very near,) both added together makes 500l. Whence it is evident, he who allows interest for discount wrongs himself considerably, which yet is very common among traders; for so much money ought to be paid, as at interest, would amount to the sum due in the time proposed.

EXAMPLE. What present money is equal to 95l. due 9 months hence, at the rebate of 5 per cent.?

See the work in the next page.

£. £. £.
As 103.75 : 100 : : 95
100

M.		£.
6	$\frac{1}{2}$	5
3	$\frac{1}{2}$	2.5
		1.25
		3.75
		100
		103.75

```
103.75)9500.00 (91.566
       93375       20
       -----     ------
        16250    11.320
        10375        12
        -----     -----
         58750    3.840
         51875        4
         -----    -----
          68750   3.360
          62250
          -----
          65000
          62250
          -----
           2750
```

£. s. d.
Answer 91 11 3 $\frac{3}{4}$

Example. What is the present value of a 10 l. bill due 4 months hence, discounted at 4 per cent.?

M.		£.
4	$\frac{1}{3}$	4
		$1.\dot{3}$
		100
		$101.\dot{3}$

As $101.\dot{3}$: 100 : : 10

```
 101      10
----     ----
91.2     1000
         1000
         -----
         900.0
```

Thus the terms are prepared for the dividing part, there being a repetend in the divisor; and the answer will produce 9l. 17s. 4d. $\frac{1}{2}$ (very near) for the present payment of 10l. due 4 months to come, discounted at 4 per cent.

Exam-

EXAMPLE. What discount must be allow'd on a bill of 94l. 10s. payable in 98 days; rebate being made at 4 ½ per cent. ?

D. £. D.
As 365 : 4.5 : : 98
4.5
490
392
365)441.0(1.2082
365
760
730
3000
2920
800
730
70

£. £. £.
As 101.2082 : 1.2082 : : 94.5
94.5
60410
48328
108738
101.2082)114.17490 (1.128
1012082
1296670
1012082
2845880
2024164
8217160
8096656
120504

1.128
20
2.560
12
6.720
4
2.880

	£.	s.	d.
Answer	1	2	6 ½

EXAMPLE. If 850l. is due at 1 year and 4 months to come, what must be discounted for the present payment, if rebate be allowed at 5 per cent.?

```
M.  |   | £.        As 106.66 : 6.66 : : 850
 4  | ⅓ | 5                          850
    |   | 1.66                   ---------
        ------                    33300
        6.66                     5328
       100                     ----------
      --------          106.66)5661.00  ( 53.075
      106.66                   53330           20
                              ---------     -------
                               32800         1.500
                               31998            12
                               -------      -------
                                80200        6.000
                                74662
                               -------
                                55380
 Answer                         53330
£.  s.  d.                     -------
53  1   6                        2050
```

N. B. In this last example I have taken in but two of the circulating decimals, and so work'd as in common, in order to compare it to the same done by the rules of repetends which brings it to the exact truth; but if the repeating figures were carried to 6 or 7 places of decimals (tho' more tedious) it would bring it to very near the same as by observing the rules of repetends.

The last example work'd by repetends.

```
As 106 : 6 : : 850
    10          6
   ---        ---
    96     9)5100
           ------
             5666
              5666
           ------
        96)5100.0 ( 53.125
           480           20
           ----       -----
            300       2.500
            288          12
           ----       -----
            120       6.000
             96       -----
           ----       £.  s.  d.
            240       53  2   6
            192       True answer.
           ----
             480
             480
           ----
```

It is evident by a preceding example, that when discount is allowed for days the operation is long and tedious, and perhaps may be one reason why tradesmen use interest for discount; but to remedy this inconveniency we have inserted the following table, which shews the discount of any sum for any number of days, at 3, 4, 5, 6, &c. per cent. interest.

	3	4	5	6	7	8	9	10
1	797978	1053741	1304631	1550788	1792344	2029427	2262159	2490660
2	1595957	2107482	2609263	3101577	2584688	4058853	4524318	4981320
3	2393935	3161222	3913904	4652365	5377032	6088280	6786477	7471980
4	3191914	4214963	5218526	6203153	7169376	8117707	9048636	9962640
5	3989892	5268704	6523157	7753942	8961721	10147133	11310795	12453300
6	4787871	6322445	7827789	9304730	10754065	12176560	13572955	14943960
7	5585849	7376185	9132420	10855518	12546409	14205987	15835114	17434620
8	6383828	8429926	10437052	12406307	14338753	16235414	18097273	19925280
9	7181806	9483667	11741683	13957095	16131097	18264840	20359432	22415940
10	7979785	10537408	13046314	15507883	17923441	20294267	22621591	24906600

Rule. Multiply the principal sum by the number of days, seek for the product in the table and that will be the discount required.

Note. If the product of the days and principal hath

7	figures, point off	4	decimals.
6		5
5		6
4		7
3		8
2		9
1		10

Exam-

EXAMPLE.

What is the diſcount of 160l. for 144 days at 5 per cent. ?

144	20000 = 2.609263
160	3000 = .3913904
	40 = .005218526
8640	
144	3.005871926
23040	

	£.	s.	d.
Diſcount required	3	0	$1\frac{1}{4}$.637

Note. By the way, the aforementioned remarks on diſcount, naturally leads me to make ſome brief conſtructions upon another erroneous method much after the ſame manner, tho' not ſo much in uſe, which is in equation of payments. The deſign of this rule is, when ſeveral ſums are due at ſeveral times, to find a mean or equated time for paying the whole debt, ſo that neither debtor or creditor may ſuſtain loſs. Now the cuſtomary rule is, to multiply each ſum by its reſpective time, and add all the products together; then divide the total by the whole debt, and the quotient is called the mean time of paying the whole debt.—But to find the juſt mean or equated time of payment, you muſt firſt find out the preſent payment of every particular ſum in the queſtion payable at a time to come, by rebating at the rate of intereſt agreed on; then find in what time the ſum of thoſe preſent worths will be augmented to the total of all the particular ſums payable at times to come according to the firſt agreement; ſo ſhall the time

found

found out, be the mean time for paying of the whole debt.—As equation of payments is very rarely made uſe of, I will leave the rule with this remark, ſo that you may prove the truth of it at your leaſure.

COMPOUND INTEREST,

OR

INTEREST UPON INTEREST,

IS that which ariſes from the principal and intereſt; that is, when intereſt on money becoming due and not paid, the ſame rate is allow'd on the unpaid intereſt as was before on the principal; ſo that the intereſt of the principal, and the intereſt of the intereſt added together, is the compound intereſt.

RULE. Multiply the principal by the rate per cent. which gives the intereſt for one year, this add to the principal, the ſum is the amount of the firſt year: Then this multiplied by the rate gives the intereſt for the ſecond year, which added to the firſt year's amount, gives the amount for the ſecond year, &c. &c.

EXAMPLE. What will 25l. amount to, if it be forborn 3 years, at the rate of 4 $\frac{1}{2}$ per cent. per annum, compound intereſt.

 See the work in the next page.

	251
	.045
	1255
	1004
	11.295
	251
1ſt year's amount	262.295
	.045
	1311475
	1049180
	11.803275
	262.295
2d amount	274.098275
	.045
	1370491375
	1096393100
	12.334422375
	274.098275
Anſwer	286.432697375 or £. 286 s. 8 d. 7 ¼

But the ſpeedieſt method to work queſtions in compound intereſt is by tables; therefore the following may ſerve as a ſpecimen.

A TABLE

A TABLE ſhewing the amount of one pound for years, at 5 per cent. per annum compound intereſt.

Years.	£.
1	1.050000
2	1.102500
3	1.157625
4	1.215506
5	1.276282
6	1.340096
7	1.407100
8	1.477455
9	1.551328
10	1.628895
11	1.710339
12	1.795856
13	1.885649
14	1.979932
15	2.078928
16	2.182875
17	2.292018
18	2.406619
19	2.526950
20	2.653298
	&c.

The method to calculate tables of compound intereſt is to raiſe the amount of 1l. to the power of the time. Thus, at 5 per cent. the amount of one pound for one year (*viz.* 1.05) being multiply'd into itſelf or rais'd to the 2d power, gives the amount of 1l. for 2 years; and that amount being multiplied into the 1ſt year's amount gives the amount for the next year; and ſo on with any other rate of compound intereſt you begin your table with.

THE

THE USE OF THE TABLE.

CASE I. To find the amount of any principal or ſum, according to the rate given at compound intereſt.

RULE. Multiply the ſum againſt the number of years propos'd by the principal, and the product will be the amount.

EXAMPLE. What will 225l. amount to in 3 years, at 5 per cent. per annum, compound intereſt?

```
        1.157625
             225
     -----------
         5788125
        2315250
       2315250
     -----------                £. s. d.
Anſwer 260.465625  or 260  9  3 3/4
```

N. B. If it is required for more years than what are in the table, take any two or more numbers as make the number of years you want, and multiply the ſums belonging to each number in the table together, which gives the ſum for that number of years; and that ſum multiply'd by the principal gives a direct anſwer.

EXAM-

A Table ſhewing the amount of 1l. for days, at 5 per cent. per ann. compound intereſt.

Days.	£.
1	1.0001337
2	1.0002674
3	1.0004011
4	1.0005348
5	1.0006686
6	1.0008023
7	1.0009361
8	1.0010699
9	1.0012038
10	1.0013376
20	1.0026770
30	1.0040182
40	1.0053612
50	1.0067060
60	1.0080525
70	1.0094009
80	1.0107511
90	1.0121031
100	1.0134569
110	1.0148125
120	1.0161699
130	1.0175291
140	1.0188902
150	1.0202531
160	1.0216178
170	1.0229843
180	1.0243527
190	1.0257228
200	1.0270949
250	1.0339825
300	1.0409164
365	1.0499999
viz.	1.05

EXAMPLE. What will be the amount of 170l. 10s. in 25 years, at 5 per cent. per annum, compound intereſt?

```
        2.653298
        1.276282
        --------
         5306596
       21226384
       5306596
     15919788
    18573086
    5306596
   2653298
   -----------------
   3.386356|478036
     170.5
   -----------------
    16931780
  23704492
  3386356
  -----------------
Anſ. 577.3736980
   or £. 577  7  5 ½
```

Suppose I wanted to know what would be the amount of 1l. for any number of days not in the table, as suppose for 194 days; then look in the table for the number against 190, and also against 4, and multiply those two numbers together, the product is the amount of 1 l. for 194 days; proceed in the same manner for any other number of days.

And suppose I would know the amount of 1l. for 5 years and 194 days, proceed thus; against 5 years is 1.276282, and the amount of 1l. for 194 days is 1.026271; the product is the amount of 1l. in 5 years and 194 days equal to 1.30981, &c.

N. B. This table of the amount of 1l. for days calculated at compound interest, ought to be made use of instead of the vulgar method in using simple interest for days, as may be seen by what follows; and as it corresponds with what has been said before on simple interest.

The simple interest of one pound is a series of terms in arithmetic progression continually increasing, but compound interest is a series of terms in geometrical progression increasing; and consequently the amount of any sum of money reckoned at simple interest will be very different from that reckon'd at compound interest whether it be for more or less than an year: See Examples.

Example for Years.

Thus 100 l. at 5 per cent. for 6 years simple interest amounts to 130l. and 100 at 5 per cent. for 6 years compound interest gives 134.00961.

Exam-

Example for days.

100 l. at 5 per cent. for 80 days ſimple intereſt amounts to 101l. 1s. 11d. and 100 at 5 per cent. for 80 days compound intereſt gives 101l. 1s. 6d. for the amount; the difference is 5d.

In order to illuſtrate this ſubject a little further, let us conſider that if a perſon pays intereſt at any leſs time than an year, he ought to have intereſt for the money paid correſponding to the time it is paid before it becomes due; for inſtance, if he pays the intereſt half yearly of 100l. at 5 per cent. in this caſe ſuch a ſum muſt be paid as if he ſhould put this intereſt money of 100l. for 6 months to uſe it wou'd gain him ſo much money, which ſums added to the intereſt money of 100l. for the other 6 months, together with the principal ſhould amount to 105l. at the year's end; and conſequently it muſt be reckoned at compound intereſt for any time leſs than an year, as well as for any time greater than an year. Now let us ſuppoſe this example to be reckon'd at ſimple intereſt, and then we ſhall ſee by the conſequence whether it amounts juſt to 105l. at the end of the year.

The intereſt of 100 l. for one day at 5 per cent. ſimple intereſt is .01369863, which multiplied by 182½ the days in half a year gives the intereſt of 100 l. for ½ an year, equal to — —

	£.	s.	d.
The intereſt of 100 l. for ½ an year, equal to — —	2	10	0
And the 2l. 10s. being put to intereſt for the other ½ year will gain	0	1	3
Now the 100l. goes on again for the other half year and conſequently gains — —	2	10	0
To which add the principal money	100	0	0
And the ſum is — —	105	1	3

Instead of 105, which is 15d. too much, and consequently must not be calculated at simple interest.

QUESTION. If a farthing was put to interest at 5 per cent. in the first year of the Christian Æra, what would it amount to if it continued out at compound interest till the year of our LORD * 1765?

The amount of 1l. for 100 years is 131.50126; which multiplied into itself gives 17292.58138 for 200 years; that sum multiply'd by itself gives 299033370.784 for 400 years; and that multiply'd by itself gives 89420956842441224.8 for 800 years; which sum multiply'd by itself gives 7996107522617726100561034842524135 for 1600 years. Now the amount of 100 years is

* It is commonly supposed that the date of the year is the number of years since the birth of Christ, and indeed it ought to be so; but it is easy to prove that we reckon 4 years too little; for the vulgar Æra of Christ's birth was fixed by Dionysius Exiguus a Roman abbot, to the end of the year of the Julian period 4713, which ought to have been fixed to the end of the 4709th year of the Julian period; and which is thus demonstrated.

Christ was undoubtedly born before the death of Herod the great; but according to Josephus (B. 17. ch. 8.) there was an Eclipse of the Moon in the time of Herod's last illness; which very eclipse our astronomical tables shew to have been in the year of the Julian period 4710 Mar. 13 d. 3 h. 21 m. after midnight at Jerusalem. Now there is reason to suppose that Christ was about an year old when Herod gave command for the children to be slain from two years old and under; for according to this command Herod concluded that he was more than one year old yet less than two; and after this command he was in Egypt,

is 131.50126, which multiplied by 23.839901 the amount of 65 years, the product will be

I 3134.977

Egypt, and all this was before the death of Herod; however supposing instead of an year old we shall suppose him only a few months old at Herod's death, even then the latest time in which we can possibly fix the Æra of his birth must be about the end of the 4709th year of the Julian period.

Again, if a man will give himself the trouble, he may find the age of Christ when he was crucified, by this means, viz. if he will find the times of all the passover full moons from the 20th to the 40th year of Christ, after the Jewish manner by adding 14 days to the time when the preceding new moon was first visible at Jerusalem; he will find that the only passover full moon which falls on a Friday in all that time is in the year of the Julian period 4746, and on the 3d day of April. Now the difference between 4709 and 4746 is 37 years for the age of Christ when he was crucified, tho' generally supposed to be only 33 years old. Now this agrees with what Phlegon an heathenish writer and others tell us, "that in the last year "of the 202d olympiad there was the most extraordinary "eclipse of the sun that ever was known, and that it was "night at the 6th hour of the day." Which agrees exactly with the time that the darkness at the crucifixion began, and therefore must have been the very same darkness, but mistaken by Phlegon for a natural eclipse of the sun, which was impossible because the moon was then in the full; so that this darkness must have been quite out of the common course of nature. And as all these accounts agree, so consequently the date of the year 1765 should be 1769; which if this had been considered in the question, and taken into the account, we should have had above 5627000000000000000000000000000 £. to be added to the amount of the 1765 years, on account of the 4 years extraordinary; and this is above 200,000 globes of fine gold each as big as the Earth that it amounts to in these last 4 years.

3134.977 for the 165 years; then that ſum multiply'd into the amount of 1600 years gives 250676131729335511175585313275117185169.9 for the amount of 1 l. for 1765 years. Laſtly, reduce 1 farthing to the decimal of a pound, viz. 960)1.0000000(.0010416; —— then multiply the amount of 1 pound for 1765 years by .0010416, and the product will be £.2609705648723535561678626834736l1602.4, anſwering to the amount of one farthing in 1765 years at 5 per cent. per ann. compound intereſt.

CASE II. *To find the amount of any annuity, penſion, &c.*

RULE. Multiply the reſpective number in the table by the annuity, ſubtract the annuity from the product, and divide by the rate; the quotient is the amount, &c.

EXAMPLE. What will an annuity of 50 l. per annum amount to in 4 years at 5 per cent. per annum, compound intereſt?

```
     1.215506
           50
   ----------
    60.775300
    50
   ----------
.05)10.775300
   ----------          £.   s.  d.
Anſwer  215.5060  or  215  10  1 1/4
```

CASE

Case III. *To find the present worth of annuities, pensions, &c.*

Rule. Divide the annuity by the tabular number, and subtract the quotient from the annuity, divide the remainder by the rate, the quotient is the present worth, &c.

Example. What is the present worth of an annuity of 30l. per annum, to continue 7 years at 5 per cent. per annum, compound interest?

```
                              30
1.4071)30.0000000 (21.320446
         28142          ——————————
         —————    .05) 8.679554
          18580        ——————————      £.  s.  d.
          14071        173.59108 or 173 11 10
          —————           Answer,
           45090
           42213
           —————
            28770
            28142
            —————
             62800
             56284
             —————
              65160
              56284
              —————
               88760
               84426
               —————
                4334
```

CASE IV. *To find the present worth of any sum.*

RULE. Divide the sum by the tabular number and the quotient is the present worth.

EXAMPLE. What is the present worth of 420l. 12s. payable 4 years hence, at the rate of 5 per cent. per annum, compound interest?

```
1.215506)420.600000(346.0287
           3646518
          ---------             £.  s. d.
            5594820       Anſ.  346  0  6 ¾
            4862024
            -------
             7327960
             7293036
             -------
              3492400
              2431012
              -------
              10613880
               9724048
               --------
                8898320
                8508542
                -------
                 389778
```

CASE V. *To find the annuity that any ſum will purchaſe.*

RULE. Add one to the number of years, take the number againſt it in the table, ſubtract the tabular number for the given years therefrom, the remainder is a dividend; then ſubtract 1 from the tabular number and divide the dividend by the remainder, the quotient multiply'd by the principal is the anſwer.

EXAMPLE. What annuity will 60l. purchaſe for 5 years continuance, at the rate of 5 per cent. per annum, compound intereſt?

```
             1.340096
             1.276282
           ----------
.276282).0638140000 ( .230974
           552564              60
          --------     ----------
           855760       13.858440
           828846
          --------
           2691400                 £.  s.  d.
           2486538      Anſwer     13 · 17  2
          --------
           2048620
           1933974
          --------
           1146460
           1105128
          --------
             41332
```

QUESTION. Suppoſe the national debt be 130 millions; what annuity will be ſufficient to pay it off in 30 years at 4 per cent. per annum, compound intereſt?

Amount of 1 l. for 30 years at 4 per cent. is —

```
                 3.2433975
                      1.04
             ----------------
                129735900
               32433975
             ----------------
               3.373133400
               3.2433975
             ----------------
2.2433975) .129735900  (  .05783
             112169875          130000000
             ------------    ---------------
              175660250      173490000000
              157037825      5783
             ------------    ---------------
              186224250  £.7517900.00000
              179471800        Annuity.
             ------------
               67524500
               67301925
             ------------
                 222575
```

So that if the national debt is 130 millions, and the ſum paid be 7517900 l. every year, that will clear the whole debt in 30 years.

Of PURCHASING FREEHOLD ESTATES.

To find the present worth of any annual rent, to continue for ever, commonly called Fee Simple.

RULE.

DIVIDE the proposed rent by the interest of 1l. for 1 year, and the quotient is the value of the estate.

Examples.

What is an estate of 200l. per annum, to continue for ever, worth in ready money, allowing the purchaser 5 per cent. per annum, compound interest?

.05)200.00

£. 4000 Answer.

A has the possession of an estate of 130l. per annum, to continue 20 years; B has the reversion of the same, from that time for ever; what must A give B, if he wou'd purchase his reversion? And what must B give A, if he would buy his possession, accounting 5 per cent. per annum, compound interest, in each case?

See the work in the next page.

2.653298)130.000000(	130
10613192	48.9956
	.05)81.0044
23868080	
21226384	1620.088
	A's possession.
26416960	
23879682	
25372780	
23879682	
14930980	
13266490	
16644900	
15919788	
725112	

	£.	s.	d.
	.05)130.00		
	2600	0	0
	1620	1	9
B's reversion	979	18	3

A,

A agrees with B for an annuity of 600l. per annum, to continue 25 years, to give him the present worth of it at 6 per cent. per annum, compound interest; but not having money enough by him, offers to make over to him a freehold estate of 12l. per annum at the same interest; what money beside will pay his purchase?

```
                           600
4.29187)600.00000 ( 139.7992              .06)12.00
        429187    ----------              ---------
        ------ .06)460.2008                  200
        1708130   ----------
        1287561    7670.013
        -------     200
         4205690  ----------
         3862683   470.013     £.  s. d.
         -------   Answer     7470  0  3
          3430070
          3004309
          -------
           4257610
           3862683
           -------
            3949270
            3862683
            -------
              865870
              858374
              ------
                7496
```

Note.

Note. If a freehold estate be purchased, and 2 per cent. allowed, it is 50 years purchase, &c. as in the following table.

Rates.	Years purchase.
2	50.
2 ½	40.
3	33.3
3 ½	28.5714
4	25.
4 ½	22.2
5	20.
5 ½	18.18
6	16.6
6 ½	15.3846
7	14.2857
7 ½	13.3
8	12.5
8 ½	11.7647
9	11.1
9 ½	10.5263
10	10.

THE

THE VALUATION OF LEASES OR ANNUITIES ON LIVES.

AS this is a neceſſary ſubject, we ought not to paſs over it in this place; and though it has been handled by many learned gentlemen, yet the theorems and rules that they have given are ſo tedious and perplexed that they can only be of uſe to perſons with an uncommon ſhare of learning; this evil we ſhall endeavour to avoid, by giving ſuch rules, that any perſon with a moderate ſhare of learning may know the value of any annuity or leaſe for lives in a very ſhort time.

And we muſt firſt obſerve, that before the value of any annuity on a life of any propoſed age can be aſcertained, it will be neceſſary to conſider how we are to eſtimate the probability of the continuance of a life for any given time; and it is evident this can be done no other way but from obſervations made from the bills of mortality for a ſeries of years together, and ſuch as are proper for the country where theſe obſervations are to be made.—In London theſe bills are drawn up by the company of pariſh clerks, and contain an account of the numbers, ages, diſeaſes, &c. of ſuch as die within the bills of mortality, that is, in London, Weſtminſter, and 10 miles round. But the great diſparity between the births and burials in London is owing to this, that the diſſenters of all ſorts baptize their children without ſending an account

count of them to the pariſh clerks, ſo that little dependance is to be had on theſe with regard to calculating annuities for life. Dr. Hally's table founded on the Breſlaw bills of mortality is of much more authority; but as theſe bills of mortality will not ſuit England *, we ſhall firſt exhibit a table of the number of perſons who have died in the ſpace of 20 years, and then give a table of the probabilities of life from Mr. Simpſon.

			Mean.
Between	0 and 2	194645	9732
	2 and 5	45984	2299
	5 and 10	19037	952
	10 and 20	16575	829
	20 and 30	40858	2043
	30 and 40	49709	2485
	40 and 50	50945	2547
	50 and 60	41311	2066
	60 and 70	32486	1625
	70 and 80	23611	1181
	80 and 90	12198	610
	90 and 100	2083	104
	100 and upwards.	181	9

The

* Although Sileſia (of which Breſlaw is the capital) is much in the ſame latitude with England, and conſequently in the ſame climate, yet it is obſerved that the ſtate of the air and weather is much more regular at Breſlaw than in England, for the frequent and ſudden change of the air from hot to cold and from cold to hot in England, eſpecially in the ſpring, beſides that moiſt damp air that we have here in winter, makes the air much more unwholeſom than at Breſlaw. Again, an Engliſhman generally makes more free with his conſtitution than a Sileſian; ſo that the tables of the value of an annuity for lives being calculated by the Breſlaw bills of mortality would give numbers too large for a purchaſe to be made in England.

The following is Mr. SIMPSON's table of the probabilities of life.

	0	1	2	3	4	5	6	7	8	9	Persons living.
Born	1280	870	700	635	600	580	564	551	541	532	
10	524	517	510	504	498	492	486	480	474	468	
20	462	455	448	441	434	426	418	410	402	394	
30	385	376	367	358	349	340	331	322	313	304	
40	294	284	274	264	255	246	237	238	220	212	
50	204	196	188	180	172	165	158	151	144	137	
60	130	123	117	111	105	99	93	87	81	75	
70	69	64	59	54	49	45	41	38	35	32	
80	29	26	23	20	17	14	12	10	8	6	

N.B. Mr. *Simpson*'s table of the probabilities of life was constructed from the London bills of mortality, but by the allowances made it differs from all others in this, that it is better adapted for country towns and villages than for large cities, as London, Paris, Pekin, &c. because it has been often observed that people in large and populous cities are more subject to mortality than in country places; and therefore on this account Mr. *Simpson*'s table ought to have the preference of all others for general use.

By the help of this table we find what probability there is for a man of a certain age, 30 for example, ſhall live 1 year; thus againſt 31 is 376 and againſt 30 is 385, the meaning of which is that out of 385 perſons living at 30, there is only 376 living at 31: Thus the probability of a perſon of 30 years of age ſhall live one year is meaſured by this fraction $\frac{376}{385}$, that is, he has the odds of 385 to 9, or nearly 43 to 1 that he does not die in one year.

So the odds that a man of 34 ſhall live 7 years is found by ſubtracting 284 in the table from 349; hence the odds is 284 to 65, or 4 $\frac{1}{3}$ to 1, that a man of 34 lives 7 years; and the like for any other number of years.

Hence it appears, that the value of inſurance upon lives ought to be regulated, there being a great difference between inſuring the life of a man of 20, and that of another of 50 years of age; ſince 'tis 65 to 1 that the man of 20 dies not in an year, and but 24 $\frac{1}{2}$ to 1 for a man of 50 years of age.

The moſt general method of valuing annuities on lives is from Mr. de Moivre; *viz.* Take the value of an annuity certain for ſo many years as are denoted by the complement of life (which is what the given life wants of 86;) multiply this value by the rate of intereſt, and divide the product by the complement of life; ſubtract the quotient from 1, divide the remainder by the intereſt of 1 pound, and this laſt quotient will expreſs the value of an annuity for any age given.

But I think we can get an eaſier rule than this of Mr. de Moivre's from the following conſideration. Suppoſe a perſon A was to receive

1l.

1 l. upon this condition, that another, B, 20 years of age shell live 1 year; quere the value of A's expectation?

Against the ages of 20 and 21 are 462 and 455; now the present value of 1 l. due at the end of one year at 5 per cent. is .95238, which multiplied by this fraction $\frac{455}{462}$ (that is, multiplying by the numerator and dividing by the denominator) gives .938 of a pound for the true value required.

In like manner the probability of a person of 20 years of age has of living 2 years is $\frac{448}{462}$, and the present worth of 1 l. due at the end of 2 years at 5 per cent. is .907; therefore multiply .907 by $\frac{448}{462}$, and it gives .8795 of a pound for the value of A's expectation to receive 1 l. at the end of the 22d year of B's life. And thus you may proceed for all the other years of his life to the extremity of age; and the sum of all these (from 20 and upwards) being found, and added together, will amount to 13 l. very nearly.

It is demonstrated by the writers on the value of lives, that the sum of the values of 1 l. for the successive years of a person's life will be found by dividing 1 by the numbers in the table of the amount of 1 l. which numbers being added together will give the value of that life which is equal to the number of these numbers so added, if there was no contingency in the case; but as we must allow for that, there arises a long and tedious algebraic operation, which is too long to be inserted here; but the rule thence arising is this:

To the value of the given life add one year's purchase, discount that sum, and multiply it by the probability of a life of one year younger than the given life, this last product will be the value of an annuity upon this life.

For example, the life of 20 being 13 l. this increased by one year's purchase is 14, which discounted at 5 per cent. is 13.3, this multiplied by the probability of a life of 19, gives 13.16 for the required value of a life of 19 years of age: And in this manner are the numbers in the following table computed, which shews the value of an annuity on any single life from 1 to 80, at the rate of 3, 4, 5 and 6 per cent. compound interest.

N. B. The numbers in the following table, and also the answers to the questions on annuities on lives shew the pounds that will purchase one pound annuity on such lives; or they shew the number of years purchase that any life or lives (contained in the table) are worth; therefore, multiply the numbers in the table, or the answers to the questions on annuities on lives by the annuity, and the product shews the value of that annuity.

The following is a table shewing the present value of an annuity of 1 l. (or, which is the same, the number of years value which such annuity is worth) for a life of any age under 81, at 3, 4, 5, and 6 per cent. compound interest.

Ages.

Ages	3	4	5	6	Age	3	4	5	6
9&10	19.0	16.4	14.3	12.7	46	12.1	10.7	9.7	8.6
8. 11	19.0	16.4	14.3	12.6	47	11.9	10.5	9.5	8.5
7. 12	18.9	16.3	14.2	12.5	48	11.8	10.4	9.4	8.4
6. 13	18.7	16.2	14.1	12.4	49	11.6	10.2	9.3	8.4
14	18.5	16.0	14.0	12.4	50	11.4	10.1	9.2	8.3
5. 15	18.3	15.8	13.9	12.3	51	11.2	9.9	9.0	8.2
16	18.1	15.6	13.7	12.2	52	11.0	9.8	8.9	8.1
17	17.9	15.4	13.5	12.0	53	10.7	9.6	8.8	8.0
4. 18	17.6	15.2	13.4	11.9	54	10.5	9.4	8.6	7.8
19	17.4	15.0	13.2	11.8	55	10.3	9.3	8.5	7.7
20	17.2	14.8	13.0	11.6	56	10.1	9.1	8.4	7.6
21	17.0	14.7	12.9	11.5	57	9.9	8.9	8.2	7.5
3. 22	16.8	14.5	12.7	11.3	58	9.6	8.7	8.1	7.4
23	16.5	14.3	12.6	11.2	59	9.4	8.6	8.0	7.3
24	16.3	14.1	12.4	11.1	60	9.2	8.4	7.9	7.2
25	16.1	14.0	12.3	11.0	61	8.9	8.2	7.7	7.1
26	15.9	13.8	12.1	10.8	62	8.7	8.1	7.6	7.0
2. 27	15.6	13.6	12.0	10.7	63	8.5	7.9	7.4	6.9
28	15.4	13.4	11.8	10.6	64	8.3	7.7	7.3	6.7
29	15.2	13.2	11.7	10.5	65	8.0	7.5	7.1	6.6
30	15.0	13.1	11.6	10.4	66	7.8	7.3	6.9	6.5
31	14.8	12.9	11.4	10.2	67	7.6	7.1	6.7	6.3
32	14.6	12.7	11.3	10.1	68	7.4	6.9	6.6	6.2
33	14.4	12.6	11.2	10.0	69	7.1	6.7	6.4	6.1
34	14.2	12.4	11.0	9.9	70	6.9	6.5	6.2	5.9
1. 35	14.1	12.3	10.9	9.8	71	6.7	6.3	6.0	5.7
36	13.9	12.1	10.8	9.6	72	6.5	6.1	5.8	5.6
37	13.7	11.9	10.6	9.5	73	6.2	5.9	5.6	5.4
38	13.5	11.8	10.5	9.3	74	5.9	5.6	5.4	5.3
39	13.3	11.6	10.4	9.2	75	5.6	5.4	5.2	5.1
40	13.2	11.5	10.3	9.1	76	5.3	5.2	5.0	4.9
41	13.0	11.4	10.2	9.0	77	5.0	4.9	4.7	4.6
42	12.8	11.2	10.1	8.9	78	4.7	4.6	4.4	4.3
43	12.6	11.1	10.0	8.8	79	4.3	4.2	4.0	3.9
44	12.5	11.0	9.9	8.7	80	3.9	3.8	3.6	3.5
45	12.3	10.8	9.8	8.6	81			3.	

We now proceed to compute the value of annuities on the longeſt lives of two lives, on the ſame principles as before; but we ſhall not give the inveſtigation of the theorems or rules, but only the rules themſelves, becauſe of the intricacy and difficulty attending it, this work being not deſign'd for Algebraiſts; and ſecondly, becauſe it would take up more room than we can poſſibly ſpare for this purpoſe.

CASE I. *If both the lives are equal.*

RULE. 1ſt. Divide the complement of a ſingle life by 6 times the amount of 1 l. for one year; 2d. ſubtract the quotient from the value of one of the lives; 3d. add half the remainder to the value of one life, and the ſum is the value of both lives.

QUEST. What is the value of the longeſt of two equal lives each of 48 years; allowing compound intereſt at 4 per cent. per annum?

Amount of 1l. for 1 year			1.04
	86		6
Life	48		6.24)38.00(6.08
Complement	38		3744
			5600
Value of a life of 48 at 4 per cent.		10.4	4992
		6.08	608
		½)4.32	
		2.16	
		10.4	
Anſwer		12.56	

CASE

CASE II. *If the lives are both unequal.*

RULE. 1ſt. Divide the complement of the oldeſt life by 6 times the amount of 1l. for one year; 2d. ſubtract the quotient from the value of the oldeſt life; 3d. multiply the remainder by the complement of the oldeſt divided by twice the complement of the youngeſt; 4th. add this product to the value of the youngeſt life, and the ſum will be the value of the longeſt of the two lives.

QUEST. What is the value of an annuity of 1 l. to continue during the life of the longeſt liver of two perſons, whoſe ages are 43 and 54; allowing 4 per cent. compound intereſt?

```
86   86                                1.04
43   54                                   6
——   ——                              ——————
43   32                          6.24)32.00(5.128
                                          3120
                                          ————
Value of the }                             800
oldeſt life. }  9.4                        624
               5.128                      ————
              ——————                      1760
               4.272                      1248
                .372                      ————
              ——————                      5120
               8544       43              4992
              29904        2              ————
             12816       ————              128
            ————————   86)32.0(.372
Value of  }  1.589184      258
the young-}  11.1          ————
eſt life. }  ————————      620
            12.689184 Anſ. 602
                           ————
                            180
                            172
                            ————
                              8
```

To find the present value of an annuity for the longest of three lives; in this is four cases.

1st CASE. *When all three lives are equal.*

RULE. 1st. Divide the complement of a single life by 6 times the amount of 1l. for one year; 2d. subtract the quotient from the value of a single life; 3d. take $\frac{3}{4}$ of the remainder and add it to the value of a single life, the sum is the answer.

QUEST. Let the three persons be each 54 years old, and compound interest be allow'd at 4 per cent. what is the present value of the longest liver?

```
86                              1.04
54                                 6
--                              ----
32                          6.24)32.00(5.128
                                  3120
                                  ----
Value of a single life  9.4        800
                        5.128      624
                        -----      ----
                        4.272      1760
                            3      1248
                        -----      ----
                     4)12.816      5120
                       ------      4992
                        3.208      ----
                        9.4         128
                        -----
Answer                 12.608
```

2d.

2d. CASE. *When the two youngest are equal.*

RULE. 1st. Divide the complement of one of the youngest lives by 6 times the amount of 1l. for one year; 2d. subtract the quotient from the value of one of the youngest lives, and take half the remainder and call it A; 3d. divide the complement of the eldest life by 6 times the amount of 1l. for one year; 4th. subtract this quotient from the value of the eldest life; 5th. multiply the remainder by the square of the complement of the eldest life, and divide by 4 times the square of the complement of one of the youngest lives; add this, and A, and the value of one of the youngest lives together, the sum is the value required.

QUEST. Let the two youngest be 34 and the eldest 54, what is the value of an annuity for the longest of the three lives, at 4 per cent.?

86		1.04	
34	1.04	6	
	6		
52		6.24)52.00(8.3	
	6.24)32.00(5.128	4992	
86	3120		
54		2080	
	800	1872	
32	624		
		208	
	1760	Value of one of the youngest lives	12.4
	1248		
			8.3
	5120		
	4992		½)4.06
	128		A 2.03

See the remainder of the work in the next page.

32	52	9.4	Value of the eldest
32	52	5.128	life.
64	104	4.272	
96	260	1024	
1024	2704	17088	
	4	8544	
		4272	
	10816		
		10816)4374.528(	.404
		43264	2 083
			12.400
		48128	
		43264	14.837 Answ.
		4864	

3d. CASE. *When the two eldest are equal.*

RULE. 1st. Divide the complement of the oldest single life by 6 times the amount of 1 l. for one year; 2d. subtract the quotient from the value of the oldest single life; 3d. multiply the remainder by 3 times the complement of the oldest single life, and divide by 4 times the complement of the youngest life; 4th. add this last number to the value of the youngest single life, the sum is the value of the longest of the three lives.

QUEST. What is the value of an annuity at 4 per cent. for the longest of 3 lives, the youngest is 20, and the two oldest are each 34?

See the work in the next page.

86 1.04
34 6
52 6.24)52.00(8.3
4992

86 2080
20 1872

52 66 208
3 4

156 264 Value of a life of 34 is } 12.4
8.3

4.06
156

2440
20333
40666

264)634.40 (2.4
528 14.8

1064 17.2 Answ.
1056

8

4th CASE. *If the lives are all unequal.*

RULE. Put A the youngest, B the next, and C the oldest life; and let *a*, *b*, and *c*, be their complement of life to 86. Then

1st. Divide *b* by 6 times the amount of 1l. for one year, subtract the quotient from the value of B's life, and multiply the remainder by *b*.

2d. Di-

2d. Divide *c* by 6 times the amount of 1 l. for one year, subtract the quotient from the value of C's life, and multiply the remainder by *c* squared, and divide by twice *b*.

3d. Add these 2 steps together, and divide the sum by twice *a*, add this to the value of A's life, and the sum will be the value of the annuity for the longest, of the 3 unequal lives.

QUEST. Let the 3 unequal lives be A 20, B 34, and C 54; what is the value of the longest, at 4 per cent.?

```
  1.04
     6
  ----
  6.24)52.00 (8.3
       4992
       ----
        2080
        1872
        ----
         208

                    Value of B's life. 12.4
                                        8.3
                                       ----
 86     86                             4.06
 34     54                               52
 ---    ---                            ----
b 52   c 32                             813
                                       20383
                                      ------
                                      211.46
```

See the remainder of the work in the next page.

6.24)32.005.(128
3120

800
624

1760
1248

5120
4992

128

52
2

104

32
32

64
96

1024

86
20

66
2

132

9.4 value of C's life.
5.128

4.272
1024

17088
8544
4272

211.46 } add.
104)4374.528 (42.0628 }
416

214
208

652
624

288
208

80

132)253.5295 (1.9206
132

1215
1188

272
264

895
792

103

14.8 value of A's life.

16.7206 Anfwer.

EXTRACTION of the SQUARE ROOT.

TO extract the ſquare root of any number, is to find out ſuch a number, as being multiply'd into itſelf, the product ſhall be equal to the number given.

All the ſingle ſquare numbers, with their reſpective roots, are contain'd in the following table.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81

1ſt. You muſt point your given number, that is, make a point over the unit's place, another upon the hundred's, and ſo upon every ſecond figure throughout.

2dly. Then ſeek in the table for the greateſt ſquare number in the firſt point towards the left hand, placing the ſquare number under the firſt point, and the root thereof in the quotient, ſubtract the ſaid ſquare number from the firſt point, and to the remainder bring down the next point, and call that the reſolvend.

3dly. Then double the quotient, and place it for a diviſor on the left hand of the reſolvend; ſeek how often the diviſor is contained in the reſolvend, (reſerving always the unit's place) and put the anſwer in the quotient, and alſo on the right hand ſide of the diviſor; then multiply by the figure laſt put in the quotient, and ſubtract the product from the reſolvend, bring down the next point to the remainder and proceed as before.

Note.

Note. If the given number be a mix'd number, viz. consisting of a whole number and a decimal together, make the number of decimal places even, that is, 2, 4, 6, &c. so that there may a point fall upon the unit's place of the whole numbers: The same method must be taken in finding the fractional part of the root, by adding cyphers by pairs to the remainder, &c.

Examples.

Let 106929 be a number given, and the square root thereof be required? As also the square root of this number 226.8741?

```
106929)327            226.8741(15.06
  9       Root.        1            Root.
------                ----
62)169                25)126
   124                   125
   ---                ---------
647)4529              3006) 18741
    4529                    18036
    ----                    -----
                              705
```

Requir'd the square root of .1024?

```
  .1024(.32
   9
  ----
62)124
   124
  ----
```

What's the ſquare root of 56078967.3?

```
 .  .  .  .
56078967.3(7488.58
49          Answer.
——
144)707
    576
   ————
1488)13189
     11904
    —————
14968)128567
      119744
     ——————
149765)882330
       748825
      ———————
1497708)13350500
        11981664
       ————————
         1368836
```

The various *Uſes* of the SQUARE ROOT.

***Firſt**, to find a geometrical mean proportional between any two numbers.*

RULE. Multiply one by the other, and extract the ſquare root of the product for the anſwer.

EXAM-

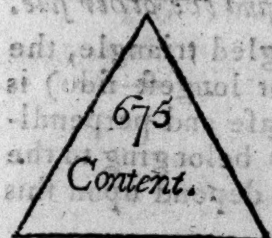
EXAMPLE. What's the mean proportional between 36 and 84?

```
   84                    3024(54·99
   36                    25       Answer.
  ----                   --
  504              104)  524
 252                     416
 ----                    ---------
 3024             1089)10800
                        9801
                        --------
                 10989)99900
                        98901
                        --------
                          999
```

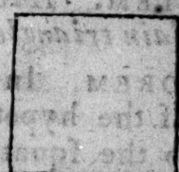
PROBLEM. *To find the side of a square that shall be equal to the content of any given superficies.*

RULE. Extract the square root of the given content for the side of a square equal thereto.

EXAMPLE.




```
      675(25·98 one side of the
      4                  square.
     ----
 45)275
    225
    ------
509)5000
    4581
    --------
5188)41900
     41504
     --------
       396
```



EXAMPLE. Admit 74685 soldiers being ordered into a square battalia, that is, having as many in rank as file?

```
   . . .
   74685(273
   4          Answer.
   ----
47)346
   329
   ----
543)1785
    1629
    ----
     156 odd men.
```

Note. If you would have a double battalia of men, extract the square root of half the number given, and that gives the number to be plac'd in the file; then there must be twice as many in rank. Or if you would have a quadruple battalia, extract the root of $\frac{1}{4}$ of the given number, &c.

PROBLEM. *Having any two sides of a right angled plain triangle given, to find the other side.*

THEOREM. In a right angled triangle, the square of the hypothenuse (or longest side) is equal to the square of the base and perpendicular. There are three cases belonging to the right angled triangle, and all depend upon this theorem.

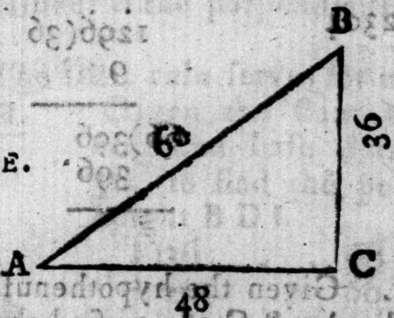
1st. When the base and perpendicular are given; add the square of the base to the square of the perpendicular, and the square root of the sum is the hypothenuse.

2d.

2d. When the base and hypothenuse are given; from the square of the hypothenuse, subtract the square of the base, and the square root of the remainder is the perpendicular.

3d. When the hypothenuse and perpendicular are given; from the square of the hypothenuse, subtract the square of the perpendicular, and the square root of the remainder is the base.

Example.



Case 1st. Given the base A C 48, and perpendicular B C 36, to find the hypothenuse ?

```
  36          48
  36          48
 ----        ----
 216         384
108         192
 ----       -----
1296         2304
             1296
            -----
             3600(60
             36      Answer.
            -----
               00
```

Case

CASE 2d. Given the baſe A C 48, and hypothenuſe A B 60, to find the perpendicular?

```
   48          60
   48          60
  ----        ----
  384         3600
  191         2304
  ----        ----
  2304        1296(36  Anſwer.
               9
              ------
            66)396
               396
            ------
```

CASE 3d. Given the hypothenuſe A B 60, and perpendicular B C 36, to find the baſe?

```
   36          60
   36          60
  ----        ----
  216         3600
  108         1296
  ----        ----
  1296        2304(48  Anſwer.
               16
              ----
            88)704
               704
            ------
```

PRO-

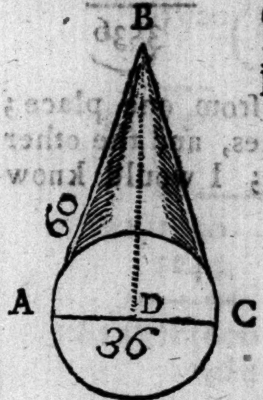
Problem.

Having any two sides of a cone, to find the other side.

By the help of the foregoing problem, you'll find, that to square the slant height (which is the length of a line drawn from its vertical point to the circumference of its base) and from that square subtract the square of the semidiameter of its base, then the square root of the remainder is the perpendicular height of the cone.

Note. The same rule serves for a pyramid.

Example. Given the slant height A B 60, and base diameter A C 36, to find the perpendicular height B D?



$\frac{1}{2}$)36
18
18
144
18
324

60
60
3600
324
3276(57.23 Ans.
25
107)776
749
1142)2700
2284
11443)41600
34329
7271

There

There is a tower whose height is 45 yards, and is surrounded by a moat 30 yards broad; of what height must a scaling-ladder be that will reach from the outside of the moat to the top of tower?

30	45	2925	(54.08 Yards.
30	45	25	Answer.
900	225	104)425	
	180	416	
	2025	10808)90000	
	900	86464	
	2925	3536	

Two men set out together from one place; one went directly north 72 miles, and the other went directly west 96 miles; I would know how far they are asunder?

72	96	
72	96	
144	576	
504	864	
5184	9216	
	5184	
	14400	(120 miles.
	1	Answer.
	22)44	
	44	
	00	

Suppose

Suppoſe a ſhip ſails 440 miles till her difference of latitude is 310 miles, I demand how far ſhe is departed from the meridian ?

```
     440                310
     440                310
   -----               ----
   17600               3100
   1760                930
   ------              -----
   193600              96100
    96100
   -------
   .  .  .  .     Miles.
   97500(312.25 very near.
   9     (  Anſwer.
   ---
 61)75
    61
    -----
 622)1400
     1244
     -----
 6242)15600
      12484
      ------
 62445)311600
       312225
       ------
```

N. B. In this queſtion, the departure is the diſtance that the ſhip is to the eaſt or weſt of the meridian, and is the baſe line ; the diſtance that the ſhip has ſailed is the hypothenuſe ; and the difference of latitude is the perpendicular, and ſhews how much the ſhip is to the north or ſouth of the place it ſet ſail from.

A

A ladder 52 ½ feet long, being so placed in a street, that it reach a window 29 feet from the ground on one side, and by turning the ladder over (without removing the foot) it will touch a moulding 40 feet from the ground on the other side; how broad is the street?

```
      52.5       29                    40
      52.5       29                    40
    ------     ----                  ----
      2625      261       2756.25    1600
     1050       58        1600
    2625       ----      --------
   -------      841       1156.25(34.0
   2756.25                 9
    841                  --------
   --------             64)256
                           256
   1915.25(43.76          --------
   16                          25
   ----
83)315
   249
   ----
867)6625              43.76
    6069              34.0
    ----             ------
8746)55600    Ans. 77.76 Feet.
     52476
     -----
      3124
```

Also

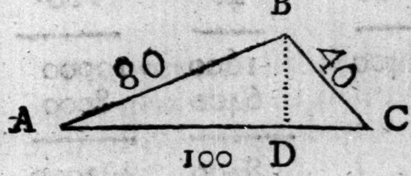
Alſo from the foregoing problem, there may be other examples very eaſily conſtructed to exerciſe the learner therein, as the three caſes all depend upon one theorem.

PROBLEM. *Having the three ſides of an oblique angled triangle given, to find the point in the baſe where the perpendicular of a right angled triangle ſhall fall.*

RULE. Square the two longeſt ſides, and from the ſum of theſe ſquares ſubtract the ſquare of the leſſer ſide, divide half the remainder by the longeſt ſide, and the quotient will give the anſwer.

EXAMPLE. Given the longeſt ſide A C 100, the next in length A B 80, and the leaſt ſide B C 40, to find the point D in the baſe where the perpendicular ſhall fall?

80	100	40
80	100	40
6400	10000	1600



10000
6400
―――
16400
1600
―――
½)14800
―――
1|00)74|00
―――
74 Anſwer.

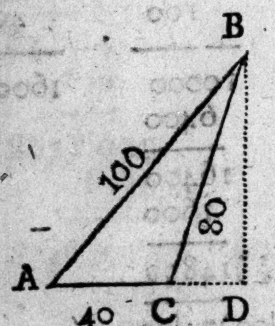
Note. To find the length of the perpendicular B D in the aforementioned triangle; from the ſquare of A B, ſubtract the ſquare of A D, and the ſquare root of the remainder is the perpendicular.

Problem.

Having the ſides of an oblique angled triangle given, to find how far a perpendicular let fall from an acute angle, will fall without the triangle.

Rule. From the ſquare of the longeſt ſide ſubtract the ſum of the ſquares of the two leſſer, and divide half the remainder by the leaſt ſide, and the quotient will be the diſtance.

Example. Given the longeſt ſide A B 100, the next in length B C 80, and the leaſt ſide A C 40, to find the point D where the perpendicular ſhall fall without the triangle?



80	40	100
80	40	100
6400	1600	10000
	6400	8000
	8000	½)2000
		4|0)100|0
		Anſwer 25

There

Example. There is a cask whose head diameter is 22 inches, bung diameter 25 inches, and the length 30 inches; what is the length of the diagonal line?

Rule. Add the ½ difference of the bung and head diameter to the head, and square the sum; add this square to the square of ½ the length, and the square root of this sum will be the diagonal required.

25	22	15 half the length.
22	1.5	15
½)3	23.5	75
1.5	23.5	15
	1175	225
	705	552.25
	470	Inches.
	552.25	777.25(27.9 very near.
		4 Answer.
		47)377
		329
		549)4825
		4941

Problem. *To find the diameter of a circle whose area shall be in a given proportion to that of a circle whose diameter is given.*

Rule.	The given diameter	multiplied / divided	by the square root of the
	intended	increase / decrease	will give the diameter of the circle.

What is the diameter of a circle whose area is 4 times as much as one of 11 inches diameter?

	11
Square root of 4 is	2
Answer	22 inches.

Admit a circle whose diameter is 63 inches; of what diameter must a circle be whose area will be 9 times less than that of the given diameter?

Square root of 9 is 3)63

Answer 21 inches.

THEOREM. All like surfaces are to each other, as the squares of their like sides, diameters, &c.

If the transverse diameter of an ellipsis be 57, and conjugate diameter 41; what will be the diameter of a circle whose area will be equal to that of the oval?

$$\begin{array}{r} 57 \\ 41 \\ \hline 57 \\ 228 \\ \hline 2337 \end{array}$$

See the remainder of the work in the next page.

```
    2337(48.34 diameter requir'd.
    16
    ----
 88)737
    704
    -----
963)3300
    2889
    -----
9664)41100
     38656
     -----
      2444
```

What will be the diameter of a circle of an equal area to a parallelogram, whose length is 26.2 inches, and breadth 19.5 inches?

```
       26.2                          650.50(25.5 in.
       19.5                          4       Answer.
       ----                          ---
       1310                       45)250
      2358                           225
      262                            ----
      ----                       504)2550
*.7854)510.9000(650.5                2520
       47124                         ----
      ------                           30
       39660
       39270
       -----
        39000
        39270
        -----
```

* The reason of this number is demonstrated in treating upon the circle in mensuration.

If a pipe of 3 inches diameter will fill a veſſel of water in an hour's time; of what diameter muſt a pipe be to fill the ſame in half an hour?

Inches.

3

3

M. M.

As 60 : 9 : : 30 (Inverſe proportion.)

9

3|0)54|0

Inches

18 (4.24

16 Anſwer.

82)200

164

844)3600

3376

224

If one fathom of cable whoſe circumference is 10 inches weighs 17lb. of what circumference muſt a cable be that one fathom thereof may weigh 10.88lb.?

Inches.

10

10

lb. lb.

As 17 : 100 : : 10.88

100

17)1088.00(64

102

68

68

64(8 inches.

64 Anſwer.

In

In the midſt of a meadow well ſtor'd with graſs,
I took juſt an acre to tether my aſs;
How long muſt the cord be, that feeding all round,
He mayn't graze leſs nor more than his acre of
ground?

5.5
5.5
———
275
275
———
30.25 ſquare yards in a perch.
160
———
181500
3025
———
4840 ſquare yds. in an acre.

.7854)4840.0000(6162.465
47124
———
12760
7854
———
49060
47124
———
19360
15708
———
36520
31416
———
51040
47124
———
39160
39270
———

6162.465(78.50 diameter.
49
———
148)1262
1184
———
1565)7846
7825
———
2150

39.25 yards.
Anſwer.

A

A legal winchester bushel is 18 ½ inches in diameter, and 8 inches deep; what must be the diameter of that bushel whose depth is 9 inches?

```
.7854          2150 the solid inches in a bushel.
    9
-------
7.0686)2150.0000(304.162
        212058
       --------
          294200
          282744
          ------
          114560
           70686
          ------
           438740
           424116
           ------
            146240
            141372
            ------
              4868

                        .  .  .  .  Inches.
                       304.1620(17.44
                       1            Answ.
                      ---
                    27)204
                       189
                    ------
                   344)1516
                       1376
                    -------
                  3484)14020
                       13936
                       -----
                          84
```

What

What must be the diameter of a cylindrical vessel of 3 feet deep, that shall hold twice as much as a vessel of 28 inches deep, and 46 inches in diameter?

```
  12              46         28
   3              46          2
  --             ---         --
  36             276         56
                184
                ----
                2116
                  56
               -----
               12696
              10580
              ------
          36)118496(3291
             108
             ---
              104
               72
              ---
              329
              324
              ---
                56
                36
                --
                200
                180
                ---
                 20
```

```
           Inches.
    3291.5(57.372
    25       Answ.
    ---
107)791
    749
    ----
1143)4255
     3429
     -----
11467)82655
      80269
      ------
114742)238655
       229484
       ------
         9171
```

Suppose

Suppose I drop a stone into a well, and it is 4 seconds before it strikes the water at the bottom, how deep is the well?

N. B. Heavy bodies fall 16.13 feet the first second, and the space they pass thro' is as the square of the time of falling.

$$\begin{array}{r} 4 \\ 4 \\ \hline 16 \end{array} \qquad \begin{array}{r} 16.13 \\ 16 \\ \hline 9678 \\ 1613 \\ \hline 258.08 \end{array}$$

Answer 258.08 feet.

Or thus.

The space passed thro' by falling bodies in each single second is as the odd numbers 1, 3, 5, 7, 9, &c. therefore in the

Sec.		
1 it goes thro'		16.13
2 it goes thro'	3 times 16.13 equal to	48.39
3 it goes thro'	5 times 16.13 equal to	80.65
4 it goes thro'	7 times 16.13 equal to	112.91
	Same depth as before	258.08

But if you would know how deep that well is which is 4 seconds between the first descent of the stone and hearing the sound it gives in striking the water at the bottom, in such case you must have recourse to the following rule.

Square 1142 (the number of feet sound flies per second) and add 4 times the product of 16.13 and 1142 multiplied by the number of seconds it is falling, extract the square root of the sum, and subtract 1142 from the root, square the

the remainder, and divide the ſquare by 4 times 16.13, and the quotient will be the true depth of the well.

1142
1142
————
2284
4568
1142
1142
————
1304164
294727.36
————
1598891.36(1264.4
1
————
22)59
44
————
246)1588
1476
————
2524)11291
10096
————
25284)119536
101136
————
18400

1264.4
1142
————
122.4

16.13
1142
————
3226
6452
1613
1613
————
18420.46
4
————
73681.84
4
————
294727.36

See the remainder of the work in the next page.

```
                    122.4
                    122.4
                    -----
                     4896
                    2448
       16.13       2448
           4      1224
       -----      ------      Feet.
       64.52 )  14981.76 ( 232.2
                12904             Depth of the last
                -----                          well.
                 20777
                 19356
                 -----
                  14216
                  12904
                  -----
                   13120
                   12904
                   -----
                     216
```

N. B. To count seconds, get a string with a bullet at the end, and make it 39 $\frac{1}{5}$ inches long, let it loose and it will vibrate seconds; that is, each vibration will be a second. Or if you have a uniform staff or stick that is 58 $\frac{4}{5}$ inches long, that will vibrate seconds also.

A

A new and expeditious method of extracting the Cube Root.

TO extract the cube root of any number, is to find such a number, as being first multiplied by itself, and then into that product, produces the given number.

RULE. 1. Point every third figure of the given number beginning at units place; then find the nearest cube to the first point; subtract, and bring the three next figures to the remainder for a resolvend.

2d. Square the quotient and multiply it by 3 for a divisor; find how many times it is contained in the resolvend, rejecting units and tens, and put the answer in the quotient.

3d. Square this new figure, and put it on the right hand of the divisor; but if the new figure should be 1, 2, or 3, then put 01, or 04, or 09 to the right hand.

4th. Multiply the last figure in the quotient by 30, and multiply it by the former figures, add this product to the divisor, and multiply the sum by the last figure in the quotient, subtract that product from the resolvend, bring down the next three figures and proceed as before.

A TABLE to find the nearest cube to the first point.

Roots.	1	2	3	4	5	6	7	8	9
Cubes.	1	8	27	64	125	216	343	512	729

85
85
425
680
7225
3
21675

EXAMPLES.

Extract the cube root of 618470208(852
512 Answer.

New Divisor 19225 / 1200 / 20425)106470
102125

New Divisor 2167504 / 5100 / 2172604) 4345208
4345208

Extract the cube root of .009142439571(.2091
8 Answ.

New Divisor 120081 / 5400 / 125481) 1142439
1129329

New divisor 13104301 / 6270 / 13110571) 13110571
13110571

Extract the cube root 82153.680(43.47
64 Answer.

5169)18153
15507

559876)2646680
2239504

56597989)407176000
396185923

10990077

I have discontinued the work for the new divisors in this and the following examples.

N. B. In carrying on the decimal part of the quotient or root, you must add triple cyphers to the resolvend, as you do double in the square root. And in the proof, add or take in the remainder.

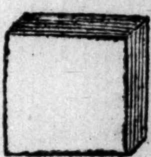
The

The various Uſes of the Cube Root.

THEOREM.

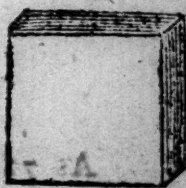
All like ſolids are to one another as the cube of their like ſides or diameters; and on the contrary, the cubes of their diameters are as their weights, &c.

There is a cube whoſe ſide is 6 feet; I demand the ſide of another cube whoſe ſolidity is double the former?



6 Feet.

```
        6
        6
      ———
       36
        6
      ———
      216
        2
      ———
      432 (7.55
      343
      ———
 15775)89000
       78875
      ———
1698775)10125000
         8493875
      ———
         1631125
```



Anſ. 7.55 Feet.

Suppoſe

Suppose the content of a globe, or cylinder, or any given solid to be 1728 inches; what is the side of a cube equal thereto?

```
  1728(12 inches.
  1    Answer.
  ----
364)728
    728
    ---
```

If an iron bullet that weighs 72 lb. is 8 inches in diameter; what will be the diameter of that bullet that weighs 9 lb.

```
            8
            8
           --
           64
            8
           --
As 72 : 512 : : 9
            9
         ----
  72)4608(64          64(4 inches.
     432              64 Answer.
     ---              --
      288
      288
      ---
```

If a cannon ball of 4 inches diameter weighs 9 lb. what ſhall be the weight of another of 6 inches diameter of the ſame metal?

```
        4              6
        4              6
      ----           ----
       16             36
        4              6
      ----           ----
As     64   :   9  ::  216
                         9
                    ------   lb.
                 64)1944  (30.375
                    192       Anſwer.
                    ----
                     240
                     192
                     ----
                      480
                      448
                      ----
                       320
                       320
                       ----
```

PROBLEM. *Having the length, breadth, depth, &c. of any ſolid, or other veſſel, to find the like dimenſions of any other ſolid or veſſel in the ſame form as the other, to be in any proportion bigger or leſſer.*

RULE. As the ſolidity of the given body is to the cube of any dimenſion, (whether length, breadth, depth, diameter, circumference, &c.)

 ſo

ſo is the ſolidity of the body requir'd to the cube of the like dimenſion, whoſe cube root will give the dimenſion itſelf which is at that operation ſought; and ſo with the reſt.

There is a ſhip of 60 tun burden, which meaſures 44 feet by the keel, middle 17.6 feet, and the depth of the hold 7.76 feet; it is required to make another ſhip of 360 tun burden of the ſame ſhape with the firſt?

Feet.
44
44
———
176
176
———
1936
44
———
7744
7744
———

	T.				T.
As	60	:	85184	: :	360
			6		

§
6|0)36|0 — 6

511104(80* feet.
512000 Keel.

§ This ſhort way of working the 1ſt. and 3d. term for a factor, anſwers the ſame purpoſe as if you multiplied by the 3d. term and divided by the 1ſt. and which method will be uſeful in many caſes.

* The true root ſhould be 79.9533, inſtead of 80, which is but about ½ an inch too long.

See the remainder of the work in the two following pages.

```
          17.6
          17.6
         ------
         1056
        1232
        176
       -------
       309.76
         17.6
       ----------
       185856
      216832
      30976
     ----------
As 60 : 5451 776  : :  360
                6
     -----------
     32710.656 (31.98
     27             Middle.
    ---------
2791)5710
     2791
    ----------
296751)2919656
       2670759
      -----------
30604924)248897000
         244839392
        -----------
          4057608
```

See the remainder of the work in the next page.

```
              7.76
              7.67
            ------
              4656
             5432
            5432
          --------
          60.2176
             7.76
          ---------
          3613056
         4215232
        4215232
        -----------
As 60 : 467.288576 : : 360
                 6
     ------------
     2803.731456 ( 14.10   Depth.
     1
     ----
 436)1803
     1744
     ------
59221)59731
      59221
      --------
      510456
```

There is a conical tun which contains 620 gallons; the bottom diameter is 80, top diameter 71.2, and depth 30 inches; it is required to make another of the same form that will hold but 140 gallons?

See the work in the next page.

Inches.
80
80

6400
80

As 620 : 512000 : 140
140

20480000
512000

620)71680000(115612.9
620

968
620

3480
3100

3800
3720

800
620

1800
1240

5600
5580

20

115612.9(48.71 Inches. Bottom diam.
64

5824)51612
46592

701329)5020900
4909303

7116531I)111597000
71165311

4043168

In the same manner is the top diameter found to be 43.35, and the depth 18.26 inches.

There

There is a cask in form of the middle frustum of a spheroid, whose head diameter is 22 inches, bung diameter 25, length 30, and content 48.3 gallons; what is the head and bung diameters and length of another spheroidical cask whose content is 80 gallons?

Inches.
22
22
————
44
44
————
484
22
————
968
968
————

As 48.3 G. : 10648 : : 80 G.
80
————
48.3)851840.0(17636
483
————
3688
3381
————
3074
2898
————
1760
1449
————
3110
2898
————
212

Inches.
17636.(26.02
8
————
1596)9636
9576
————
20295604)60000000
40591208
————
19408792

Anſwer 26.02 inches for the head diameter; and proceed in the ſame manner for the bung diameter and length.

How many 3 inch cubes can I have cut out of a cube whoſe ſide is 15 inches?

```
                15
                15
               ---
                75
               15
              ----
              225
               15
              ----
             1125
 3            225
 3           ----
--        27)3375(125 Anſwer.
 9           27
 3           --
--            67
27            54
              ---
              135
              135
              ---
```

Or thus,

```
3 ) 15
    --
     5
     5
    --
    25
     5
   ---
   125
```

Suppoſe

Suppose a globe of 18 inches diameter; how many globes each of 4 inches diameter is equal thereto?

```
      4)18
      ----
       4.5
       4.5
      ----
       225
      180
      ----
      20.25
        4.5
      -----
      10125
      8100
      -----
Answer 91.125
```

A farmer borrow'd of his neighbour a cube of hay whose side was 8 feet, and pay'd back again 2 cubic pieces whose side was 4 feet each; quere, whether the lending farmer was fully paid?

```
     4            8
     4            8
    --          ---
    16           64
     4            8
    --          ---
    64      128)512(4
     2          512
    --          ---
   128
```

That is, 128 is but $\frac{1}{4}$ of 512, and consequently the farmer paid but $\frac{1}{4}$ of what he borrow'd.

DUODE-

DUODECIMALS
OR
CROSS-MULTIPLICATION,

IS a method among workmen of conceiving every ſuperior place to be divided into twelves, and is of uſe only in caſting up the contents of their ſuperficial and ſolid works.

Moſt artificers take the dimenſions of their work in feet, inches, and parts, (viz. 12 parts;) others in feet, inches, and half quarters, or 8 parts; otherwiſe inſtead of 8 parts they take quarters, and reckon every quarter as 3 parts; ſo that theſe actually follow the duodecimal ſcale, altho' they ſeldom take notice of any other parts of an inch, than 3, 6, or 9, or give and take (as they call it) for the intermediate ones. And as in decimals, the places deſcend by tens from the units place to the right hand, ſo in duodecimals, the decreaſe is by twelves from the feet place to the right hand, and the ſame are noticed by feet, inches, parts, ſeconds, thirds, &c.

Obſerve if you multiply the length and breadth together, it gives the ſquare or ſuperficial content; and that product multiplied by the depth, gives the ſolidity. The eaſieſt method for multiplying duodecimally may be obtained by obſerving the following rule.

Firſt write down the feet, inches, &c. (to be multiplied together) in ſuch order, ſo that the denominations ſtand under thoſe of the ſame

name, viz. feet under feet, inches under inches, &c. drawing a line underneath; then multiply each term in the multiplicand (beginning at the highest) by the feet in the multiplier, and write each result under its respective term: In the same manner multiply the multiplicand by the inches in the multiplier, and write the result of each term one place more to the right hand than before. Also observe the same method in multiplying by parts, setting the result of each term still one place more to the right hand than in the last operation, &c. and the sum of these results gives the product required.

N. B. Always when the multiplicand has feet, the first result of the multiplier (if you begin at the feet in the multiplicand) will fall directly under the multiplier and be of the same name; which method must be more feasible, and not so liable to err as the common method of beginning to multiply the lowest denomination of the multiplicand first, and carrying by twelves every time.

Also in the practical customary method, when the feet in the given dimensions are many, that to multiply them by the lesser denominations and take a twelfth of their product, will each time require some work to be done on a spare paper; but the method I have laid down will enable any one to perform the operations with more ease and less charge to the memory; for there will be no occasion for any separate work in this new method, except in adding the numbers up at last, and carrying the 12's to the next superior place. (But should the number of feet run very high, the operation may be better done by decimals than any way else.)

Ex-

Examples.

If a board be 12 $\frac{1}{2}$ feet long, and 15 inches broad, how many ſquare feet doth it contain ?

```
          F.    I.
          12 —  6
           1 —  3
          ————————
          12 —  6
                36 — 18
   ———————————————————
Anſwer    15 —  7 —  6
```

If a board is 16 — 7 by 1 — 8, how many feet ?

```
          F.      I.      F.    I.
          16 —    7  by   1  —  8
           1 —    8
          ———————————————————————
          16 —    7
                128  —   56
   ——————————————————————————
Anſwer    27 —    7  —    8
```

Suppoſe a board to be 8 F. 6 I. and a half long, and broader at one end than the other, and the mean * we'll ſuppoſe to be 19 Inches and $\frac{1}{4}$; what the content ?

```
          F.   I.   P.
          8 —  6 —  6
          1 —  7 —  3
          ——————————————————
          8 —  6 —  6
              56 — 42 — 42
                   24 — 18 — 18
   ————————————————————————————
Anſwer   13 —  8 —  5 —  1 —  6
```

* Thus in order to find a mean between the two ends of an oblong ſurface, ſuch as a board being one end

What does a marble flab come to, whofe length is 5 feet 3 inches, and breadth 1 foot 9 inches and 4 parts, at 6s. 6d. per foot?

F.	I.	P.		In.			s.	d.
1 —	9 —	4		4	$\frac{1}{3}$		6	6
5 —	3							9
5 —	45 —	20				2	18	6
	3 —	27 —	12				2	2
9 —	4 —	0 —	0	£.		3	0	8 Anf.

If an oak beam be 1 F. 6 I. by 1 F. 3 I. and 18 F. 3 I. long, how many folid feet are there?

	F.	I.		
	1 —	6		
	1 —	3		
	1 —	6		
		3 —	18	
	1 —	10 —	6	
Length	18 —	3		
	18 —	180 —	108	
		3 —	30 —	18
Solidity	34 —	2 —	7 —	6 Anfwer.

broader than the other (having no regard to the thicknefs, or if you have it muft be the fame thro' the whole,) here you add both ends together and take half the fum for a mean breadth; but this method will not ferve for the mean of an unequal folid, fuch as the fruftum of a pyramid, &c. for then you muft take the geometrical mean of the two ends, &c.

If

If a beam be 1 F. 5 I. 6 P. by 1 F. 1 I. 10 P. and 16 F. 3 I. 7 P. long, how many ſolid feet?

	F.	I.	P.				
	1	5	6				
	1	1	10				
	1	5	6				
		1	5	6			
			10	50	60		
	1	8	2	1	0		
Length	16	3	7				
	16	128	32	16			
		3	24	6	3		
			7	56	14	7	
Solidity	27	4	9	7	5	7	
	Anſwer.						

N.B. In ſome ſorts of work the anſwer will be requir'd in ſquare yards; then you are to divide the number of ſquare feet by 9 (the ſquare feet in a yard) and the quotient will be ſquare yards.

How many ſquare yards are there in a piece of work which meaſures 12 F. 7 I. 3 P. by 10 F. 5 I. 4 P.? And what will it come to at 3s. 3d. per yard?

See the work in the next page.

	F.		I.		P.				
	12	—	7	—	3				
	10	—	5	—	4				
	120	—	70	—	30				
			60	—	35	—	15		
					48	—	28	—	12
	9)131	—	7	—	8	—	8	—	0
	Yds.		F.		I.		P.		S.
Content	14	—	5	—	7	—	8	—	8

d.		
3	$\frac{1}{4}$	14
		3
		42
		3 6
		2 0
		2|0)4|7 6
		£. 2 7 6 Cost.

F.		s. d.
3	$\frac{1}{3}$	3 3
1	$\frac{1}{3}$	1 1
1	-	$4\frac{1}{4}$
		$4\frac{1}{4}$
		$2\frac{1}{2}$ for the odd inches, &c.
		2 0

Here you see when the number of feet runs pretty high, that the only difficulty is in adding up the numbers at last; and then the rule may be facilitated by having a table of the 12's contained in a pretty large number, by heart. Besides, this as well as all other rules of the like nature, may be aided by the rules of practice, &c. And where the dimensions contain but a few feet, you'll find this new method the most certain and most agreeable of any yet practised, except you take the dimensions by decimals.

PLANI-

PLANIMETRY,

OR

MENSURATION OF SUPERFICIES.

SUperficial figures are all ſuch as have only length and breadth, not having any commenſurable thickneſs.

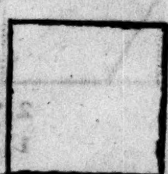
PROBLEM I.

To find the area of a geometrical Square, or rectilineal Parallelogram.

RULE. Multiply the baſe by the perpendicular altitude (viz. the length by the breadth,) and the product will be the area or ſuperficial content.

EXAMPLE. Let the ſide of a ſquare be 3 feet 10 inches; what's the content in feet, inches, &c.?

F. I.
3 10



	F.		I.		
	3	—	10		
	3	—	10		
	9	—	30		
			30	—	100
Anſwer	14	—	8	—	4

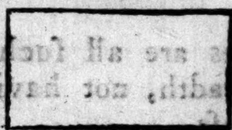
EXAM-

EXAMPLE. Suppose a square in form of a parallelogram, 18 feet long, and 9.5 feet broad, what's the content?

By decimals.

Feet.
18
9.5
———
90
162
———
Facit 171.0

18 Feet.



9.5

.5
12
—
6.0

By cross-multiplication.

F. I.
18 — 0
9 — 6
———
162 — 0
108 — 0
———
Facit 171 — 0 — 0

The same rule holds for a rhombus or rhomboides: since 'tis evident from the two next figures that all parallelograms upon the same bases and altitude are equal.

A Rhombus.

1.67

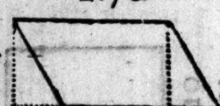
1.45

1.67
1.45
———
835
668
167
———
Area 2.4215

A Rhomboides.

2.70

1.46



2.70
1.46
———
1620
1080
270
———
Area 3.9420

From

From these figures it will not be difficult to conceive, that the area of every triangle is the $\frac{1}{2}$ of the parallelogram that circumscribes it; therefore the rule for obtaining the area of any triangle, is to multiply the base by the perpendicular altitude, and take the $\frac{1}{2}$ of that product for the area: Or if you multiply the base by half the perpendicular; or multiply half the base by the whole perpendicular; and any of these three ways will give the content.

An equilateral Triangle.

1.9
1.6

114
19

$\frac{1}{2}$)3.04

Area 1.52

An acute Triangle.

2.45
.85

1225
1960

2.0825 Area.

$\frac{1}{2}$)1.70

.85

An obtuse Triangle.

$\frac{1}{2}$)1.70

.85

3.92
.85

1960
3136

Area 3.3320

A right angled Triangle.

2.03
.9

1.827
Area.

$\frac{1}{2}$)1.8

.9

Observe

Observe, any angle greater than a right angle is obtuse, and that which is less is acute.

From the foregoing problem and corollaries likewise flow the following rule for finding the area of a trapezium and all irregular polygons.

To find the area of a Trapezium.

First divide it into two triangles by a diagonal line drawn from the two most acute angles, and let fall two perpendiculars upon that diagonal line from the two remaining angles; then multiply half of that diagonal by the sum of the two perpendiculars, and the product will be the area required.

Or if you add the two perpendiculars together, and take half the sum and multiply it into the diagonal, that will give the content; or you may find the areas of the two triangles, and add those two areas together for the content.

A Trapezium.

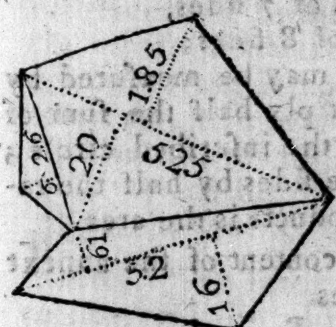
$$\begin{array}{r} 2.53 \\ 1.45 \\ \hline 3.98 \\ 2.5 \\ \hline 1990 \\ 796 \\ \hline \text{Content}\quad 9.950 \end{array} \qquad \begin{array}{r} \tfrac{1}{2})5.0 \\ \hline 2.5 \end{array}$$

To

To find the content of any irregular figure, with more ſides than a trapezium.

Divide it into trapeziums or triangles or what elſe it will bear; ſo find the different areas ſeverally, and add them together for the content.

For example; give up the content of the following irregular figure.



1.85
2.0

$\frac{1}{2}$)3.85

1.925
5.25

9625
3850
9625

10.10625
5.746
.78

Area 16.63225

.61
1.6

$\frac{1}{2}$)2.21

1.105
5.2

2210
5525

5.7460

$\frac{1}{2}$)2.6

1.3
.6

.78

PROBLEM

PROBLEM II.

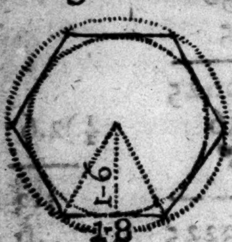
To find the area of any regular Polygon.

A regular polygon hath all its ſides equal, and is ſuch as can be inſcribed in a circle, ſo that all the angular points of it may be in the periphery or circumference of that circle which conſtitutes it. There are innumerable kinds, but the moſt common ones are theſe,

viz. a { Pentagon of 5 ſides, Hexagon of 6 ſides, Heptagon of 7 ſides, Octagon of 8 ſides;

All which or any other may be meaſured by the following rule: Multiply half the ſum of the ſides by the radius of the inſcribed circle; or multiply the ſum of the ſides by half the radius, and either of the products is the area.

EXAMPLE. Find the content of the annext hexagon in feet and inches.



	F.	I.	
	1 —	8	
		3	
	5 —	0	
	1 —	6	
	5 —	0	
		30	— 0
Content	7 —	6	

PROBLEM III.

To find the area of any Circle.

A circle may be conceived as made up of an infinite number of acute triangles whoſe baſes are

are infinitely ſmall and coincide with the periphery of the circle, and whoſe vertices meet in the center; and the laſt rule for regular polygons will ſerve for obtaining the area of the circle; viz. Multiply the radius of any circle (which is half the diameter) by half the periphery of the circle, and the product is the area.

Now by ſeveral operations and tedious proceſſes, the peripherys are found in proportion to the diameters of circles very near, as 3.1416 to 1. Hence it appears that the area of that circle whoſe diameter is multiply'd by half its circumference is equal to .7854; and the area of every circle bears proportion to the ſquare of its diameter as .7854 does to 1; ſo that if you multiply the ſquare of any circle's diameter by .7854, the product will give the area required.

Note. The ſame rules ſerve for a ſemicircle, or quadrant, by taking the parts accordingly.

A Circle.

½)11.12	½)3.54
5.56	1.77
1.77	
3892	
3892	
556	
9 8412 Area.	

Again.

```
      3.54
      3.54
    ------
      1416
     1770
    1062
    ------
   12.5316
     .7854
  --------
    501264
   626580
 1002528
877212
---------
9.84231864 Area.
---------
```

Again, by cross-multiplication.

```
                                   F.   I.   P.
 .12            .54             ½)3  —  6  —  5
  12             12               ----------------
 -----          ----              1  —  9  —  2
1.44 inches.   6.48
 .12             12
 -----          ----
5.28 parts.    5.76

                        F.    I.    P.
                        5  —  6  —  8
 F.    I.    P.         1  —  9  —  2
½)11 — 1  —  5         -------------------------
 ----------------       5  —  6  —  8
  5  — 6  —  8         45  — 54  — 72
                             10  — 12  — 16
                       -------------------------
               Area 9 —  9  —  7  —  1  —  4
                    -------------------------
```

Here

Here follows several *useful propositions* about a Circle.

The diameter of a circle being given, to find the circumference.

As 1 is to 3.1415927, so is the diameter to the circumference.

The circumference being given to find the diameter.

As 3.1416 is to 1, so is the circumference to the diameter. Multiply Circumf. by .3183

The circumference being given to find the area.

As 1 is to .0795775, so is the square of the circumference to the area required.

By the area of a circle to find the diameter.

As 1 is to 1.2732395, so is the content to the square of the diameter.

By the area of a circle to find the side of a square equal to it.

Extract the square root of the area.

By the area of a circle to find the circumference.

As 1 is to 12.5663706, so is the area to the square of the circumference.

Having the circumference of a circle to find the side of the inscribed square.

As 1 is to .2250791, so is the circumference to the side inscribed.

Having the area of a circle to find the area of the inscribed square.

As 1 is to .6366197, so is the area of the circle drawn about the square, to the inscribed square.

PROBLEM IV. *Of an Ellipsis.*

An ellipsis or oval bears the same proportion to its circumscribing parallelogram as the circle does to its circumscribing square. Therefore multiply the transverse by the conjugate diameter, and that product again by .7854 for the area.

Transverse diameter 4.35
Conjugate diameter 3.04

1740
13050

13.2240
.7854

528960
661200
1057920
925680

Area 10.38612960

PROBLEM V. *Of a Parabola.*

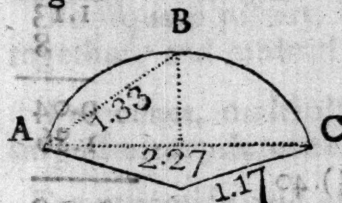
A parabola is $\frac{2}{3}$ of the parallelogram circumscribing it. Therefore multiply the greatest ordinate by $\frac{2}{3}$ of the corresponding abscissa, and the product is the area required.

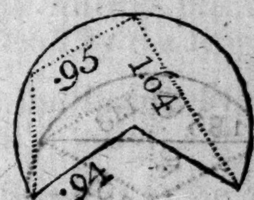
1.71	2.53
2	1.14
3)3.42	1012
1.14	253
	253
	Area 2.8842

Problem VI. *Of a Sector.*

A sector is equal to a triangle whose base is equal to the sectoral arch, and perpendicular altitude equal to the radius which describes that arch. Therefore multiply the radius by half the arch-line for the area of the sector.

But first to find the length of the arch-line, multiply the chord of half the segment A B by 8, and from the product subtract the chord of the whole segment A C, divide the remainder by 3, and the quotient is the arch-line A B C sought.





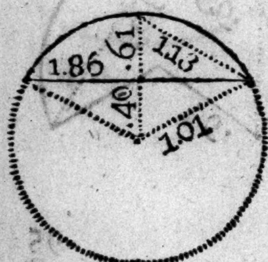
1.33	.95
8	8
10.64	7.60
2.27	1.64
3)8.37	3)5.96
2.79 Arch Line.	1.986
½)2.79	.94
1.395	7946
1.17	17880
9755	2d. Area 1.86746
1395	
1395	
1st. Area 1.63205	

N. B. There being two ſuch-like arch-lines in the 2d. ſector as in the 1ſt. in that caſe you take the whole inſtead of half.

Problem VII.

To find the area of the ſegment of a Circle.

Take the difference between the area of the ſector under the ſame arch with the ſegment, and the area of the triangle under the chord and radius, for the area of the ſegment.



½).40
.2

1.86
.2
.372

1.13
8
9.04
1.86
3)7.18
½)2.393
1.196
1.01
1196
119666
1.20863
.372
Area .83663

Note. It may ſometimes happen that you may meet with mixt or compound figures, ſuch as are compoſed of rectilineal and curvileneal figures

figures together: To find the area of ſuch, you muſt firſt find the area of the ſeveral figures of which the whole compound figure is compoſed, and add all the areas together, the ſum will be the area of the whole compound figure.

Some general rules for finding the ſuperficial content of round or ſquare figures, &c.

For round pillars, multiply the length by the circumference.

For ſquare pillars, add the ſides or breadths together, and multiply the total by the length.

For cones, multiply half the length by the largeſt circumference.

For pyramids, add all the breadths at the baſe together, and multiply half the length by the total.

For globes, multiply the area of the greateſt circle by 4, it gives the content.

STERE-

STEREOMETRY,

OR

MENSURATION OF SOLIDS,

TEacheth how to meaſure and give up the content of all ſolid bodies, which are ſuch as do conſiſt of length, breadth, thickneſs, &c.

PROBLEM I.

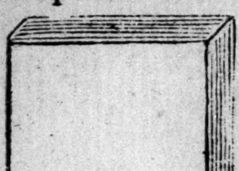
To find the ſolidity of any Cube, Priſm, or Cylinder.

A priſm, &c. is a ſolid body conſtituted of an infinite ſeries of equal areas, that of its baſe being one of the terms, and its height the number of all the terms; hence therefore may be deducted the following rule.

Multiply the area of the baſe by the depth or perpendicular altitude, and the product ſhall be the ſolid content.

Note. In the following examples, the ſolid figures are formed more to anſwer the purpoſe they are deſign'd for, than for perſpectives; as alſo there is this advantage, that the learner may very eaſily form an idea to make others by them.

Example of a Cube.



1.9 Feet.

Feet.
1.9
1.9
171
19
3.61
1.9
3249
361
Solid content 6.859

Admit the following a ſquare Priſm, viz. more in length than the cube; what's the content in feet, inches, &c.?

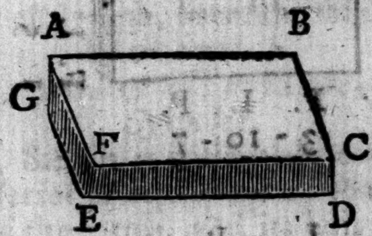
F. L. P.
1 5 5

F. I. P.
3 - 10 - 7

	F.	I.	P.				
	1 —	5 —	5				
	1 —	5 —	5				
	1 —	5 —	5				
		5 —	25 —	25			
			5 —	25 —	25		
	2 —	1 —	3 —	4 —	1		
Length	3 —	10 —	7				
	6 —	3 —	9 —	12 —	3		
		20 —	10 —	30 —	40 —	10	
			14 —	7 —	21 —	28 —	7
Anſw.	8 —	2 —	1 —	6 —	7 —	2 —	7

Let

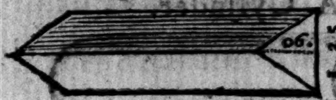
Let A B C D E F G represent a Parallelopipedon, whose length A B is 3.88 feet, breadth B C 1.86, and depth 1.04 feet; what's the solid content?



Feet.
1.86
1.04
————
744
1860
————
1.9344
3.88
————
154752
154752
58032
————
Facit 7.505472

A

A triangular Prism.

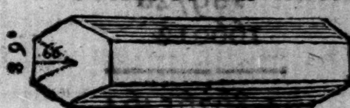


$\frac{1}{2}$).90

·45

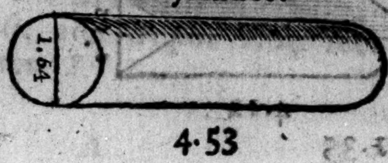
```
      1.35
       ·45
      ----
       675
      540
    ------
     .6075
      4.12
    ------
     12150
     6075
    24300
  --------
  2.502900 content.
```

A Prism of six sides.




```
      .68
      .66
     ----
      408
     408
    -----
    .4488
        3 half the sides.
   ------
   1.3464
     3.85
   ------
    67320
  107712
  40392
 --------
 5.183640 content.
```

A Cylinder.




```
      1.54
      1.54
    ------
       616
      770
     154
  --------
    2.3716
     .7854
  --------
     94864
   118580
   189728
  166012
  ----------
  1,86265464
        4.53
  ----------
    558796392
   931327320
  745061856
  ------------
8.4378255192 solid content.
```

The

The same by cross-multiplication.

	F.	I.
½)	4 —	10
	2 —	5

The dimensions in feet, inches, &c. thus; 4 feet 10 inches in circumference, 1 foot 6 inches and 6 parts diameter, and depth 4 feet 6 inches and 5 parts.

	F.	I.	P.			
½)	1 —	6 —	6			
	0 —	9 —	3			
	2 —	5				
	0 —	18 —	6			
		0 —	45 —	15		
	1 —	10 —	4 —	3		
	4 —	6 —	5			
	4 —	40 —	16 —	12		
		6 —	60 —	24 —	18	
			5 —	50 —	20 —	15
Content	8 —	5 —	4 —	5 —	3 —	3

The customary method of measuring circular solids is to girt them about the middle with a string, and take ¼ of that girt for the side of a square, by which they measure the piece as if it was square; but it is easy to be demonstrated that the fourth part of the circumference of a circle cannot be equal to the side of a square of equal area to that circle, it being less. For in-

 stance,

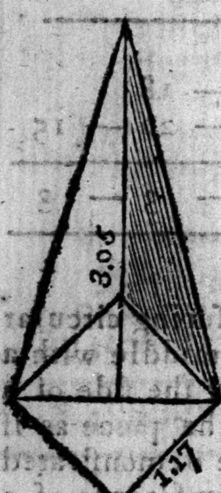
ſtance, in the laſt example, the 4th part of the circumference is 1 foot, 2 inches and an half, which being ſquared and multiplied by the length will give a ſmall matter under 6 feet 8 inches for the content; and thus you ſee the difference of the two methods.

PROBLEM II.

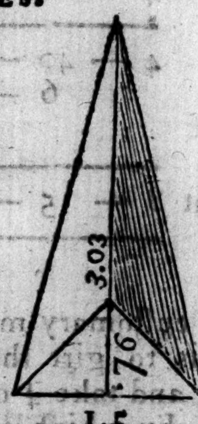
To find the content of any Pyramid or Cone.

A pyramid is $\frac{1}{3}$ of its circumſcribing priſm, and a cone is $\frac{1}{3}$ of its circumſcribing cylinder. Therefore, multiply the area of the baſe by $\frac{1}{3}$ of the perpendicular altitude, and the product will be the content: Or 'twill be the ſame thing if you multiply the area of the baſe by the whole height, and take $\frac{1}{3}$ of the product for the content.

EXAMPLES.



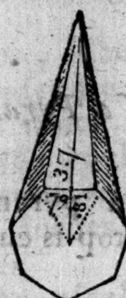
1.17
1.17
——
819
117
117
——
1.3689
3.05
——
68445
410670
——
$\frac{1}{3}$)4.175145
Content 1.391715

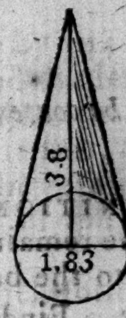


1.5
.38
——
120
45
——
.570
1.01
——
570
5700
Content .57570

$\frac{1}{2}$).76
——
.38

3)3.03
——
1.01





A Cone.

$\frac{1}{2}$).81
.405
.79
3645
2835
.31995
7
2.23965
3.7
1567755
671895
$\frac{1}{3}$)8.286705
2.762235 content.

1.83
1.83
549
1464
183
3.3489
.7854
133956
167445
267912
234423
2.63022606
.38
2104180848
789067818
$\frac{1}{3}$)9.994859028

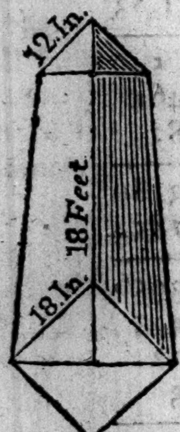
Content of the cone 3.331619676

Problem III.

To find the content of the frustum of a Pyramid or Cone.

Definition. A frustum of a pyramid, &c. is the remaining part when the top is cut off parallel to the base.

Rule. Find the area of both the greater and lesser ends; then find a geometrical mean betwixt those areas, which three numbers add together, and multiply by $\frac{1}{3}$ of the depth for the content.



12
12
———
144

18
18
———
144
18
———
324
144
———
1296
1296
324
———
46656(216
4
———
41)66
41
———
426)2556
2556
———

$\frac{1}{3}$)18
———
6

216
324
144
———
144 { 12)684
———
12)57
———
4.75
6
———
Content in feet 28.50

Note. The reaſon of dividing by 144 is, that the baſes are taken in inches, and the length in feet; which if all had been taken in inches, and you would have the anſwer in feet, the laſt operation would be to divide the reſult by the ſolid inches in a foot, inſtead of the ſquare inches.

The uſual way to meaſure ſuch ſolids, is to take a ſquare about the middle of the piece, which they take to be a mean ſquare: This way when the piece is pretty near as thick at one end as at the other, is ſomething near the truth; but when there is a great diſproportion between the ends of the piece, the error is conſiderable, and falls ſhort of the true content.

Of the fruſtum of a Cone.

The rule is the ſame in effect as the fruſtum of a pyramid, but the following may prove ſomewhat ſhorter for the fruſtum of a cone than the former; viz. To the rectangle or product of the diameter of the two baſes, add the ſquares of the ſaid diameters, and multiply the ſum by .7854, the product will be the triple of a mean area, which multiply'd by $\frac{1}{3}$ of the perpendicular height will give the ſolid content.

See the work in the next page.

9	18	18
9	9	18
81	162	144
	324	18
	81	324

567
.7854
2268
2835
4536
3969

144 { 12)445.3218
12)37.11015

3.0925125
4.75
154625625
216475875
123700500

Solid content 14.689434375

$\frac{1}{3}$)14.25
4.75

The common method used in measuring these solids, is to girt them about the middle between both ends, and take a 4th of the compass and estimate it as though it was the side of a square, as in the remark on the cylinder. The method is best adapted to the measuring of timber trees, and round timber for sale, and seems chiefly design'd for that purpose, as something confi-

derable

derable ought to be allowed for waste, &c. (beside the bark) before such timber can be fit for use; but in other matters where it is required to be more exact, the loss is great.

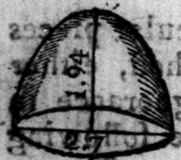
The greatest error is, that all circular pieces are in general measured by this method, either on account of its ease, or through ignorance in knowing no truer way: But let the following remark explain the nature of this customary method used in rough timber for sale; that is, measurers and workmen reckon 40 feet of unhewn or rough timber to the load, supposed to weigh about a ton, or 20 hundred weight: For as hewn timber is measured by the square, it is near exact; but rough timber is measured by the girt, or quarter compass, and is about $\frac{1}{5}$ less than exact. Therefore in buying of timber, it amounts to much the same, whether it be measured by the quarter compass at 40 feet solid to a load, or measured exact at 50 feet to a load.

PROBLEM IV.

To find the content of a Parabolic Conoid.

RULE. Multiply the area of the base by half the height for the solidity.

See the work in the next page.




```
                    2.7
                    2.7
                  -----
                    189
                    54
                  -----
 ½)1.94             7.29
   ----            .7854
   .97            ------
                    2916
                   3645
                  5832
                 5103
                --------
                5.725566
                     .97
                --------
               40078962
              51530094
             ----------
Content      5.55379902
```

Note. A parabolic conoid is $\frac{1}{2}$ of its circumscribing cylinder; and an hyperbolic conoid (being much in the same form) is $\frac{5}{12}$ of its circumscribing cylinder: The difference lies in the curve towards its base, that of the hyperbolic kind being less curved than the other as it approaches its base, and therefore falls off from its center, having the outside lines more straight.

PROBLEM V.

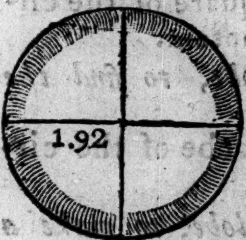
To find the content of a Sphere, &c.

A sphere or globe is a round solid body, and is $\frac{2}{3}$ parts of its circumscribing cylinder. Therefore, multiply the area of that circle whose diameter is equal to the globe's axis with $\frac{2}{3}$ of the globe's axis for the content. Or if you multiply the cube of the diameter by .5236, that will

will give the solid content. Otherwise, multiply the axis or diameter into the circumference, the product is the superficial content; which multiplied by a sixth part of the axis, gives the solidity.

A spheroid bears the same proportion to its circumscribing cylinder as the sphere does; therefore multiply the area of the conjugate axis by $\frac{2}{3}$ of the transverse, and the product is the content of the spheroid.

A Sphere.




```
      1.92
      1.92
      ----
       384
     1728
     192
    ------
    3.6864
      1.92
    ------
     73728
   331776
   36864
  --------
  7.067888
     .5236
  --------
  42407328
 21213664
14135776
35339440
------------
3.7007561568
```

Content of the sphere.

A Spheroid.


```
 2.1
 2
----
3)4.2
----
 1.4
```

```
        1.27
        1.27
        ----
         889
        254
       127
      ------
      1.6129
       .7854
      ------
       64516
      80645
     129032
    112903
  ----------
  1.26677166
         1.4
  ----------
   506708664
  126677166
 -----------
 1.773480324
```

Content of the spheroid.

Here

Here follows several *useful propositions* about the Sphere.

Having the axis of a globe, or diameter, to find the superficial content.

As 1 is to 3.1415927, so is the square of the axis to the superficial content.

By the circumference of a globe, to find the superficial content.

As 1 is to .31832, so is the square of the circumference to the superficial content.

By the circumference of a globe, to find the solid content.

As 1 is to .016887, so is the cube of the circumference to the solid content.

Having the solid content of a globe, to make a cube equal to the globe.

Extract the cube root out of the solid content of the globe.

By the axis of a globe, to make a cube equal to the solid content thereof.

As 1 is to .80604, so is the axis to the cube root, &c.

By the circumference of a globe, to make a cube equal to the solid content.

As 1 is to .256556, so is the circumference to the cube root, &c.

By the axis of a globe, to make a square equal to the superficial content of the globe.

As 1 is to 1.77254, so is the axis to the square root, &c.

PRO-

PROBLEM VI.

To find the content of the segment of a Sphere.

RULE. To three times the square of half the segment's base, add the square of the height or versed sine; multiply this sum by the versed sine, and that product by .5236; and this last product is the content.

$\frac{1}{2}$)2.64

1.32
1.32

264
396
132

1.7424
3

5.2272
.4489

5,6761
.67

397327
340566

3.802987
.5236

22817922
11408961
7605974
19014935

Content 1.9912439932

.67
.67

469
402

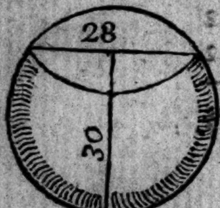
.4489

PRO-

Problem VII.

To find the content of the frustum of a Sphere.

Rule. To the area of the frustum's diameter, add the area of the depth, more one third of the same area; multiply the sum by half the depth for the content.




```
      28
      28
     ---
     224
     56
     ---
     784
   .7854
  ------
    3136
   3920
  6272
 5488
---------
 615.7536
⅓)706.86
  235.62
---------
1558.2336
       15
---------
 77911680
15582336
---------
Content 23373.5040
```

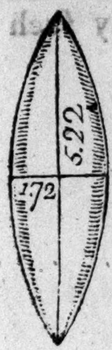
```
      30
      30
     ---
     900
   .7854
  ------
706.8600
```

```
½)30
  --
  15
```

PROBLEM VIII.

To find the content of a Parabolic Spindle.

A parabolic ſpindle is $\frac{8}{15}$ of its circumſcribing ſpindle: Therefore, multiply the area of the diameter with $\frac{8}{15}$ of the greateſt ordinate for the content.




```
        5.22
           8
    ---------
15)41.76(2.78
   30
   ------
   117
   105
   ------
    126
    120
   ------
      6
```

```
              1.72
              1.72
         ---------
               344
             1204
             172
         ---------
            2.9584
             .7854
         ---------
            118336
           147920
          236672
         207088
         ---------
        2.32352736
              2.78
         ---------
       18588821888
      1626469152
      464705472
         ---------
Solid content 6.4594060608
```

N. B. The ſolidity of any irregular body may be determined by immerſing the ſame in a veſſel of water; for the ſolid content of the additional ſpace occupied by the fluid on account of the immerſed body, will be equal to the ſolidity of that body; and thus may the ſolidity of ſtatues, &c. be very exactly determin'd.

Here follows some various Examples *in* MENSURATION.

There is a court whose length is $21\frac{1}{2}$ feet, and breadth $17\frac{1}{2}$, which is to be pav'd with ſtones each 18 inches ſquare; how many ſuch ſtones will pave it?

```
              21.5
       1.5    17.5
       1.5    ----
      ----    1075
        75   1505
       15    215
      ----   ----
  2.25)376.25(167 Anſwer.
       225
      -----
       1512
       1350
      -----
        1625
        1575
      -----
          50
```

If

If a board be 19 inches broad, how much in length will make a ſquare foot?

```
       12
       12
      ——   Inches.
  19)144(7.579
     133   Anſwer.
     ———
      110
       95
      ———
      150
      133
      ———
       17
```

How many ſquare rods are there in a wall which is $87\frac{1}{4}$ feet long, $12\frac{1}{2}$ feet high, and $2\frac{1}{2}$ bricks thick?

Note. Brickwork is generally eſtimated by the rod ſquare, which contains $272\frac{1}{4}$ ſquare feet; but for practice 272. Alſo bricklayers compute or value their work at the rate of a brick and an half thick; and if the thickneſs of the wall ſhould be more or leſs than ſuch, it is reduced to that thickneſs by multiplying the area of the wall, by the number of half bricks the thickneſs of the wall is of, and dividing the product by 3.

See the work in the next page.

```
          87.25
           12.5
        -------
          43625
         17450
         8725
        -------
272)1090.625 (  4.009
    1088           5
    --------    ------
        2625  3)20.045
        2448    ------
        -----   6.6816 Answer.
         177    ------------
```

To find how much in length makes a solid foot of any squared or regular timber, (i. e.) *such as have equal bases.*

RULE. Always divide 1728 (the solid inches in a foot) by the area of the base; the quotient is the length of the foot.

If a piece of timber be 22 inches deep, and 15 inches broad, how much in length will make a solid foot?

```
                 Inches.
        330)1728( 5.23
            1650   Answer.
            -----
  22         780
  15         660
  ---        ----
  110        1200
  22          990
  ---        ----
  330         210
```

Admit

Admit a ſquare piece of timber in form of the fruſtum of a pyramid 12 feet long, the ſide of the greater baſe 21 inches, and the ſide of the leſſer baſe 3 inches; how far muſt be meaſur'd from the greater end to cut off 5 ſolid feet?

10.5 half the greater ſide.
1.5 half the leſſer ſide.

—— F.

As 9 : 12 : : 10.5
12

9)126.0

14 } Feet the whole length of the pyramid.

21
21

21
42

441
14

1764
441

$\frac{1}{3}$)6174

144)2058(14.3 Solid content of the whole pyramid (very near.)
144 5

618 9.3 The remaining part of the whole pyramid when the 5 ſolid feet are cut off from the greater end.
576

420
432

See part of the work in the next page.

14
14
56
14
196
14
784
196

As 14·3 : 2744 : : 9·3
9·3
8232
24696

14·3)25519·2(1784·559
143
1121
1001
1209
1144
652
572
800
715
850
715
1350
1287
63

See the remainder of the work in the next page.

```
 .  .   .(14.
1784.559(12.129
1         ------
-----      1.871
364)784
    728
    -----
43561)56559
      43561
      ------
4399564)12998000
         8799128
         --------
441010521)4198872000
F.        3969094689
Answer 1.871         ----------
     12   229777311
-----------
viz. 22.452 inches.
-----------
```

N. B. I have not any where treated of giving up the content by Scale and Compasses, nor have I as yet made any mention of the Sliding-Rule, the treating of which I have reserved for the following article of Gaging; having each example work'd both by pen and sliding-rule; so that when a person comes to understand the nature of the slide, he may make use of it in any other practical case as well as in gaging; for the improvements on the slide are now become very useful, and are far more expeditious and convenient than that antique method of computing the content by scale and compasses.

GAGING.

G A G I N G.

THAT gaging is included in the ſcience of Stereometry is very plain to be underſtood, as the capacities or contents of all ſorts of veſſels for liquors, &c. are computed as tho' they were really ſolid bodies.

And herein I preſuppoſe that the learner is already well acquainted with the principal rules of arithmetic, eſpecially in multiplication and diviſion, both in whole numbers and decimal parts; and he ſhould likewiſe underſtand the rule of proportion. Alſo he ought to have ſo much ſkill in ſolids as to determine at ſight what ſort of figure any utenſil is of, or what figure, &c. it may be reduced to, ſo that its dimenſions may be taken, and the content thereof computed as near as poſſible; for there is not any tun or caſk, &c. ſo regularly made, as by the rules of art 'tis requir'd to be. Likewiſe the learner muſt know, that all dimenſions in gaging are taken in inches and decimal parts of an inch; for the content of a gallon or buſhel, &c. is known by the ſtandard of its kind to be a certain number of cubic inches, &c. which I ſhall further acquaint my pupil with. And he muſt alſo know that in the buſineſs of gaging, all ſuperficies or areas are always underſtood to be one inch deep, otherwiſe it could not be ſaid (as in the gager's language it is,) that the area of ſuch a ſquare, or ſuch a circle, &c. is ſo many gallons, &c.

Note. The barrel in the exciſe is reckon'd at 34 gallons, both for ale and beer.

I ſhall

I shall first begin with finding the areas of superficies in ale and wine gallons, and in malt bushels.

EXAMPLE 1st. Suppose a square (which has all its sides equal,) and the side 45.5 inches; what will its area be in ale and wine gallons, and in malt bushels?

RULE. Multiply the length or breadth (being here equal) into itself, and the product will be the area in inches; then divide by the cubic inches in the respective gallon, &c. and the quotient will be the area requir'd.

Cubic inches. { 282 in a ale gallon. 231 in a wine gallon. 2150 in a malt bushel.

```
       45.5
       45.5
      -----
       2775
      2275
     1820
     -------
282)2070.25(7.34
    1974
    ----
      962
      846
     ----
      1165
      1128
      ----
        37
```

See the remainder of the work in the next page.

231)2070.25(8.96 Wine.
1848

2222
2079

1435
1386

49

2150)2070.25(.96 Malt.
19350

13525
12900

625

The same by the Sliding-Rule.

Set each proper divisor upon A to a side of the square on B, then against the other side of the square on A is the area on B:

		A		B		A		B	
Viz.	As	282	:	45.5	: :	45.5	:	7.34	ale gall.
	As	231	:	45.3	: :	45.5	:	8.96	wine ga.
	As	2150	:	45.5	: :	45.5	:	.96	of a malt bushel.

EXAMPLE 2d. Let the length of a tun, back, cooler, cistern, couch, &c. be 145.3 inches, and mean breadth 88.6 inches; what is the area in ale, wine, and malt?

RULE. Multiply the length by the breadth, and divide by the cubic inches, &c.

See the work in the next page.

```
          145.3
           88.6
        -------
           8718
          11624
         11624
        -------
282)12873.58(45.65
    1128        Ale.
    ----
     1593
     1410
     ----
      1835
      1692
      ----
       1438
       1410
       ----
         28

231)12873.58(55.72
    1155        Wine.
    ----
     1323
     1155
     ----
      1685
      1617
      ----
       688
       462
       ---
       226

2150)12873.58(5.98
     10750       Malt.
     -----
      21235
      19350
      -----
       18858
       17200
       -----
        1658
```

Same

Same by the Slide.

Set each proper divisor on A to the length on B, then against the breadth on A is the area on B:

		A		B		A		B	
Viz.	As	282	:	145.3	: :	88.6	:	45.65	ale.
	As	231	:	145.3	: :	88.6	:	55.72	wine.
	As	2150	:	145.3	: :	88.6	:	5.98	malt.

Note. When one length or breadth will not serve, find a mean length or breadth thus; viz. take them in several places, and divide the total by the number of times taken.

Example 3d. If the length of the base of a triangle be 60 inches, and its perpendicular height 36 inches, what will its area be in ale, wine, and malt?

Rule. Multiply the base into half its perpendicular, and divide by the cubic inches, &c.

```
½)36
 ———
  18
  60
 ———
282)1080(3.8
     846   Ale.
    ————
    2340
    2256
    ————
      84

231)1080(4.6
     924   Wine.
    ————
    1560
    1386
    ————
     174

2150)1080.0(.5
     10750   Malt.
     —————
        50
```

By

By the Slide.

Set each proper divisor on A to the base on B, then against half the perpendicular on A is the area on B:

		A		B		A		B	
Viz.	As	282	:	60	: :	18	:	3.8	ale.
	As	231	:	60	: :	18	:	4.6	wine.
	As	2150	:	60	: :	18	:	.5	malt.

Note. By the same rule, you may easily find the area of any trapezium, or polygon, &c. First divide it into triangles, and then find the areas of those triangles in inches; the sum of those areas being divided by its proper divisor, will give the area requir'd. But if it be a trapezium, one operation may serve.

EXAMPLE 4th. Suppose the diameter of a circular vessel to be 45.5 inches; what is the area in ale, wine, and malt?

RULE. Divide the square of the diameter by these circular divisors, viz. 359 for ale, and 294 for wine gallons; and 2737.47 for malt bushels.

See the work in the next page.

45.5
45.5
———
2275
2275
1820
———
359)2070.25(5.76
1795 Ale.
———
2752
2513
———
2395
2154
———
241

294)2070.25(7.04
2058 Wine.
———
1225
1176
———
49

2737.47)2070.2500(.75
1916229 Malt.
———
1540210
1368735
———
171475

By the Slide.

Set each of the above diviſors on A to the diameter on B, then againſt the ſaid diameter on A is the area on B:

		A		B		A		B	
Viz.	As	359	:	45.5	: :	45.5	:	5.76	ale.
	As	294	:	45.5	: :	45.5	:	7.04	wine.
	As	2737.47	:	45.5	: :	45.5	:	.75	malt.

EXAMPLE 5th. Let the tranſverſe diameter of an ellipſis be 55 inches, and conjugate diameter 45 inches; what's the area in ale, wine, and malt?

RULE. Multiply the tranſverſe diameter by the conjugate, and divide by the circular diviſors.

See the work in the next page.

55
45
―――
275
220
―――
359)2475(6.8
2154 Ale.
―――
3210
2872
―――
338

294)2475(8.4
2352 Wine.
―――
1230
1176
―――
54

2737.47)2475.000(.904
2463723 Malt.
―――
1127700
1094988
―――
32712

By the Slide.

Set each proper divisor on A to the transverse diameter on B, then against the conjugate diameter on A is the area on B:

		A		B		A		B	
Viz.	As	359	:	55	: :	45	:	6.8	ale.
	As	294	:	55	: :	45	:	8.4	wine.
	As	2737.47	:	55	: :	45	:	.904	malt.

The following Examples are to compute the contents of all solid bodies used in gaging.

EXAMPLE 6th. Suppose a cube whose every side is 24 inches; what's the content in ale, wine, and malt?

RULE. Cube the side and divide by the square divisors.

See the work in the next page.

24
24
———
96
48
———
576
24
———
2304
1152
———
282)13824(49
1128 Ale.
———
2544
2538
———
6

231)13824(59.8
1155 Wine.
———
2274
2079
———
1950
1848
———
102

2150)13824(6.42
12900 Malt.
———
9240
8600
———
6400
4300
———
2100

The same by the Slide.

Upon the *Rule* you'll find these square gage-points on the side D, being the square roots of the solid inches in each respective gallon or bushel, viz. 16.79 for ale; 15.19 for wine; and 46.36 for malt. Then set each gage-point on D to 24 on C, and against 24 on D is the content on C:

	D		C		D		C	
Thus	As 16.79	:	24	: :	24	:	49	ale gall.
	As 15.19	:	24	: :	24	:	59.8	wine gall.
	As 46.36	:	24	: :	24	:	6.42	malt bush.

EXAM-

EXAMPLE 7th. Admit a prism in form of a parallelopipedon as follows; 45.6 inches in length, breadth 27.5 inches, and depth 21.5 inches; what's the content in ale, wine, and malt?

RULE. Multiply the length by the breadth, and that product by the depth, and divide by the square divisors.

```
     45.6
     27.5
    -----
    2280
   3192
   912
  -------
  1254.00
    21.5
  -------
   6270
  1254
 2508
--------
282)26961.0(95.6
    2538       Ale.
    ----
     1581
     1410
     ----
      1710
      1692
      ----
        18
```

```
231)26961.0(116.7
    231        Wine.
    ---
     386
     231
     ----
     1551
     1386
     ----
      1650
      1617
      ----
        33

2150)26961.0(12.54
     2150      Malt.
     ----
     5461
     4300
     -----
     11610
     10750
     -----
       8600
       8600
       ----
```

To work the ſame by the Slide.

Firſt find a mean proportional betwixt the length and the breadth thus; ſet either of the two numbers given upon D to the ſame on C, then againſt the other number on D is the mean requir'd on C;

	D		C		D		C	
viz. As	27.5	:	27.5	: :	45.6	:	35.4	mean length.

Then ſet each proper gage-point for a ſquare to the depth on the lines C and D, and againſt your mean is the content.

		D		C		D		C	
Thus	As	16.79	:	21.5	:	35.4	:	95.6	ale.
	As	15.19	:	21.5	:	35.4	:	116.7	wine.
	As	46.36	:	21.5	:	35.4	:	12.54	malt.

Or otherwiſe you may find the content upon the ſide of the *Rule* A and B; viz. after finding the area, ſet 1 on A to the area on B, then againſt the depth on A is the content on B.

		A		B		A		B	
Thus	As	282	:	45.6	: :	27.5	:	4.44	areas.
	As	231	:	45.6	: :	27.5	:	5.42	
	As	2150	:	45.6	: :	27.5	:	.58	
Then	As	1	:	4.45	: :	21.5	:	95.6	contents.
	As	1	:	5.42	: :	21.5	:	116.7	
	As	1	:	.58	: :	21.5	:	12.54	

Note.

Note. From the foregoing examples it will be obvious how to find the content of any ſquare ſolid or regular priſm with more or leſs ſides; viz. to multiply the area of the baſe by the depth and divide by the ſquare diviſors, &c.

Now upon the bottom of the *Rule*, under the lines A and B, is a reverſe line, mark'd M D, ſignifying malt depth; ſo that if you ſet the length upon M D to the breadth on B, then againſt the depth on A is the content on B.

EXAMPLE. Let the length of a couch be 255 inches, breadth 161, and depth 7.5; what's the content in buſhels?

	MD		B		A		B	
As	255	:	161	: :	7.5	:	143.2	Anſwer.

EXAMPLE 8th. Suppoſe a veſſel in form of a cylinder, whoſe diameter is 38.7 inches, and the depth 59.5 inches; what's the content in ale, wine, and malt?

RULE. Multiply the ſquare of the diameter by the depth, and divide by the circular diviſors.

See the work in the next page.

38.7

```
          38.7
          38.7
         -----
          2709
         3096
        1161
       -------
       1497.69
          59.5
       -------
        748845
       1347921
       748845
       -------
359)89112.555(248.2
    718              Ale.
    ----
    1731
    1436
    ----
     2952
     2872
     ----
      805
      718
      ---
       87—
```

```
294)89112.5(303.1
    882          Wine.
    ---
      912
      882
      ---
       305
       294
       ---
        11
```

```
2737.47)89112.555(32.5
        821241        Malt.
        ------
         698845
         547494
         ------
         1513515
         1368735
         -------
          144780
```

The same by the Slide.

Upon the *Rule*, on the ſide D, is the following circular gage-points, (which are the ſquare roots of their proper diviſors) viz. 18.94 for ale, 17.14 for wine, and 52.32 for malt. Then ſet each proper gage-point on D to the depth on C, and againſt the diameter on D is the content on C: Thus

		D		C		D		C	
Thus	As	18.94	:	59.5	: :	38.7	:	248.2	ale.
	As	17.14	:	59.5	: :	38.7	:	303.1	wine.
	As	52.32	:	59.5	: :	38.7	:	32.5	malt.

EXAMPLE 9th. Admit a tun or veſſel, &c. in form of the fruſtum of a cone, thus; the greater diameter 40 inches, the leſſer diameter 30, and the depth 60 inches; what's the content in ale gallons?

RULE. Find the area of each baſe, and a mean proportional between them, multiply the ſum of thoſe three by one third part of the depth or height, and the product is the content.

30	40
30	40
359)900(2.506	359)1600(4.456
718 Leſſer area.	1436 Greater area.
1820	1640
1795	1436
2500	2040
2154	1795
346	2450
	2154
	296

See the remainder of the work in the next page.

```
        4.456
        2.506
      -------
        26736
      222800
      8912
      -------
      11.166736(3.341          1/3)60
       9                          ---
      ---                          20
      63)216
         189                    2.506
      ------                    4.456
      664)2767                  3.341
          2656                 ------
      --------                 10.303
      6681)11136                   20
           6681                ------
           ----           Ans. 206.060 ale gall.
           4455
```

The same by the Slide.

	D	C		D	C	
As	18.94 :	1	: :	30 :	2.506	lesser area.
As	18.94 :	1	: :	40 :	4.456	greater ar.
As	2.506 :	2.506	: :	4.456 :	3.341	mean proportional.

```
  10.303
      20
  ------
Ale gallons 206.060
```

EXMA-

EXAMPLE 10th. Suppoſe the dimenſions of a tun or ciſtern in form of the fruſtum of a pyramid, as follows; the length of the greater baſe 96 inches, and its breadth 65.3; the length of the leſſer baſe 75.8, and its breadth 42.7 inches; and the depth 40.35 inches; what's the content in malt buſhels?

```
       65.3
       96
     ------
      3918
     5877
     ------
2150)6268.8(2.915
     4300   Greater
     ------   area.
     19688
     19350
     ------
       3380
       2150
       ------
       12300
       10750
       ------
        1550
```

```
            75.8
            42.7
          ------
            5306
           1516
           3032
          ------
2150)3236.66(1.505
     2150        Leſſer
     ------        area.
     10866
     10750
     ------
      11660
      10750
      ------
        910
```

See the remainder of the work in the next page.

```
      2.915
      1.505
    -------
      14575
     145750
     2915
    -------
    4.387075(2.094
    4           Mean.
    -----
 409)3870
     3681
    ------
4184)18975
     16736
    ------
      2239

     6.514                1/3)40.35
     13.45                  -------
    ------                   13.45
     32570
    26056                    2.915
   19542                     1.505
   6514                      2.694
   ------                   ------
Answer 87.6130 bushels.      6.514
```

The same by the Slide.

	D		C		D		C
As	46.36	:	65.3	: :	96	:	2.915 greater area.
As	46.36	:	75.8	: :	42.7	:	1.505 lesser area.
As	2.915	:	2.915	: :	1.505	:	2.094 mean.

This sum multiply'd by $\frac{1}{3}$ of the depth gives the content as before. } 6.514

Note.

Note. If you wou'd gage a globe, or oval solid, or the segment or frustum of a globe, &c. you may work as in mensuration; they being of little use in practical gaging.

A practical rule for finding the content of any conical tun, copper, by-tub, or cask.

Take the diameter somewhat above the middle, if it be broader at top than at bottom, or on the contrary, somewhat next to the broadest end; and thus it is reduc'd to a cylinder: Then multiply the square of the diameter by the depth, and divide by the circular divisors, &c.

The following is the practical method of inching, gaging, and fixing any tun, copper, &c. in order for the ready casting up of each gage.

Whether the sides are straight or not from top to bottom, you are to take your several areas as follows; that is, the area to each diameter taken in the middle of every 10 inches from the bottom upwards; thus, the first diameter at 5 inches from the bottom, the second at 15 inches, &c. and each of these areas serve for 10 inches one above the other: A specimen of which is in the next page.

Depth	43.5	Areas
Diameters	41.4	4.77
	43.8	5.34
	40.6	4.54
	36.0	3.60

See the content upon every inch of the depth, (as fixed in the dimension books of excise.)

Gall.

Whole content is 199.195

Depths	Contents	Depths	Contents
1	3.6	23	97.42
2	7.2	24	102.76
3	10.8	25	108.10
4	14.4	26	113.44
5	18.0	27	118.78
6	21.6	28	124.12
7	25.2	29	129.46
8	28.8	30	134.80
9	32.4	31	139.57
10	36.0	32	144.34
11	40.54	33	149.11
12	45.08	34.	153.88
13	49.62	35	158.65
14	54.16	36	163.42
15	58.70	37	168.19
16	63.24	38	172.96
17	67.78	39	177.73
18	72.32	40	182.50
19	76.86	41	187.27
20	81.40	42	192.04
21	86.74	43	196.81
22	92.08	43.5	199.195

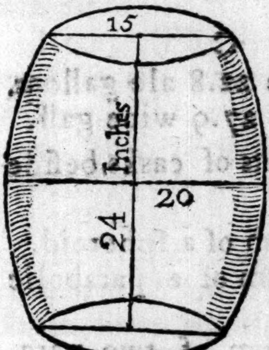
OF

Of CASK GAGING.

To gage a cask in form of the middle frustum of a spheroid, or those much curved.

Rule. First take the difference betwixt the bung and head diameter, and multiply it by 7 tenths, adding the same to the head, and it will reduce the cask to a cylinder: Then multiply the square of the diameter by the length, and divide by the circular divisors for the content.

Give up the content of the following example of a spheroidical cask, in ale and wine gallons.




```
    20
    15
  ----
     5
    .7
  ----
   3.5
    15
  ----
  18.5
  18.5
  ----
   925
 1480
 185
------
342.25
```

```
     342.25
         24
     ------
    136900
    68450
     ------
359)8214.00(22.88 Ale.
    718
    ----
    1034
     718
    ----
     3160
     2872
     ----
      2880
      2872
      ----
         8
```

See the remainder of the work in the next page.

 294)

```
294)8214.00(27.93
    588        Wine.
    ----
    2334
    2058
    ----
     2760
     2646
     ----
      1140
       882
      ----
       258
```

The same by the Slide.

	D		C		D		C	
Thus	As 18.94	:	24	: :	18.5	:	22.8	ale gallons.
	As 17.14	:	24	: :	18.5	:	27.9	wine gall.

N. B. There are three sorts of casks beside the conical ones, &c.

Viz.
- 1st. The middle frustum of a spheroid.
- 2d. The middle frustum of a parabolic spindle.
- 3d. The middle frustum of two parabolic conoids abutting upon one common base.

There are rules for finding the content of each sort severally; but in general practice, and particular that of the excise, such casks are respectively taken in the 1st. variety (as is represented by the *figure* in the preceding page) so that one general rule may serve for bulged casks.

If you wou'd gage each ſort ſeverally, the neareſt method is to multiply the difference of the bung and head diameters by .7 for the firſt form, which are thoſe much curved; by .65 for the ſecond form, if the ſtaves be ſomewhat leſs curved; and by .6 for the third form, if the ſtaves between the bung and head be very little curved; then add the product to the head diameter, and the ſum is a mean diameter.

N. B. When you meet with casks repreſenting the lower fruſtums of two equal cones, you may either multiply the difference of the bung and head diameters by .55 as above, or you may gage them as fruſtums, &c. as in the 9th example.

Note. The diagonal line of the ſpheroidical cask in the preceding example will be 21.219 inches. Thus another cask in the ſame form will hold 78 gallons wine meaſure if the dimenſions are as follows; viz. head diameter 24.286 inches, bung diameter 28.334, length 31.225, and diagonal 35.839 inches. Then if you would find the content of any cask in the ſame form by the diagonal line only, obſerve this proportion; As the cube of 35.839 is to 78 gall. ſo is the cube of any given diagonal to the content of its cask. For example; Suppoſe a cask whoſe diagonal is 53.84; ſay, as the cube of 35.839 is to 78 gall. ſo is the cube of 53.84 to 408 wine gallons for anſwer; and ſo with any other cask of like form.

Again, to gage casks by the Sliding-rule.

Upon the ſide of the *Rule* is plac'd a line of inches, and under it three other lines ſignifying the three varieties, the uſe of which is to reduce a cask to a cylinder.

RULE. Firſt ſubtract the head from the bung diameter, and find the difference on the ſaid line of inches; add that number (which you find againſt ſuch difference) to the head diameter and 'twill reduce the cask to a cylinder: Then ſet the gage-point to the length, and againſt the mean diameter is the content.

EXAMPLE. Given the head diameter 24.5 inches, bung diameter 29.7, and length 45, to find the content in ale and wine gallons?

As 18.94 : 45 : : 28.14 : 99.7 ale gallons.
As 17.14 : 45 : : 28.14 : 121.2 wine gallons

Another Example by the Rule.

Let the head be 19.6, bung diameter 24.5, and length 28.2; what's the content in wine and ale gallons?

As 17.14 : 28.2 : : 23.03 : 50.8 wine gallons.
As 18.94 : 28.2 : : 23.03 : 41.6 ale gallons.

Of

Of Ullaging *of* Casks.

Firſt to ullage a ſtanding caſk, being part full.

Rule. Divide the wet or dry inches by the caſk's length, and if the quotient exceeds .5000, add to the ſaid quotient one tenth part of the exceſs, but if it be under .5000, ſubtract one tenth part of the want; ſo will the ſum or remainder be a decimal number, by which if you multiply the content of the veſſel, the product will be the quantity of liquor therein if your dividend was the wet inches, but the quantity of liquor which is required to fill it up if the dividend was the dry inches.

Obſerve in ullaging, that there muſt be 4 decimal places at leaſt in the quotient.

Example. Suppoſe the caſk's content to be 61 gallons, whoſe length is 32 inches, 18 inches wet, and 14 dry; what will be the contained liquor, and alſo the vacuity?

```
32)18.0000 (  .5625
   160        .00625 one 10th of the exceſs.
   ———        ——————
   200        .56875
   192            61
   ———
    80         56875
    64        341250
   ———        ——————
   160        34.69375 contain'd liquor.
   160        ——————————
   ———
```

See the remainder of the work in the next page.

	.5000
32) 14.0000	.4375
128	
	.0625
120	
96	.00625 one 10th of the want.
240	.43125
224	61
160	43125
160	258750
	26.30625 vacuity.

34.69375
26.30625

Proof 61 gall.

To work the same by the Sliding-rule.

Upon the same side of the *Rule* C and D are two lines mark'd with segment standing and segment lying. First set the bung's diameter on C to 100 on segment standing, and against the wet inches on C is a 4th number, which keep: Secondly set 100 upon the line B to the cask's content on A, then against the number last found you'll find the quantity of liquor contained.

N. B. By the same rule you'll find how much it wants to fill it up if you make use of the dry inches.

To

To find the ullage of a cask which lies on one side.

RULE. Divide the wet or dry inches by the bung's diameter; if the quotient be above .5000, add to it $\frac{1}{4}$ part of the excess, and multiply that sum by the cask's content for the liquor in the cask; but if it be under .5000, take from it one fourth part of what it wants, and multiply the remainder by the cask's content for the liquor which is wanted to fill it up.

EXAMPLE. Let the cask be the same as before only in another position, and not so full of liquor; content 61 gallons, bung diameter 28 inches, wet 7 inches, and dry 21; what is the vacuity, and contained liquor?

28)21.0000 (.7500
196 .0625

140 .8125 .7500
140 61 .5000

00 8125 $\frac{1}{4}$).2500
48750

.0625

Vacuity 49.5625

28)7.0000 (.2500
56 .0625

140 .1875 .5000
140 61 .2500

00 1875 $\frac{1}{4}$).2500
11250

.0625

Liquor in the cask 11.4375

To

To perform the ſame by the Slide, follow the directions of the former rule, only make uſe of the length inſtead of the bung, &c.

Note. In ullaging of casks, the content of the cask is always made uſe of, whether it be in ale or wine gallons.

To find the content of any hogſhead or cask of dry goods in feet, inches, &c.

RULE. Take a fourth part of the girt of the bulge, and multiply it by itſelf, which multiply by the length of the veſſel, and to the product add the one fifth thereof, and the ſum is the content.

EXAMPLE. Let the cask be 7 feet long, and 9 feet girt, how many feet?

	F.	I.	
$\frac{1}{4}$)	9 —	0	
	2 —	3	
	2 —	3	
	4 —	6	
		6 —	9
	5 —	0 —	9
			7
$\frac{1}{5}$)	35 —	5 —	3
	7 —	1 —	0, &c.
Facit	42 —	6 —	3, &c.

SURVEYING OF LAND

IS the application of the rules of Planimetry to the measuring and planing the areas of real fields, grounds, &c. by the help of proper instruments.

In the following instructions for surveying of land, we shall make use of no other instrument but the Chain, for that used properly is the most compleat instrument for the purpose; far exceeding the best plain-tables, theodolites, circumferentors, &c. both in accuracy and expedition, whether the surveys are large or small, from a single field to a lordship, county or kingdom. Therefore the surveyor being furnished with a chain only, he may proceed after the following methods, which a little practice will make familiar to him.

N. B. The chain is 4 poles or 22 yards in length; it is divided into 100 decimal parts or links, each link contains 7.92 inches. And in an acre is contained 10 square chains, (viz. 10 in length and 1 in breadth;) or there are 100000 square links in an acre.

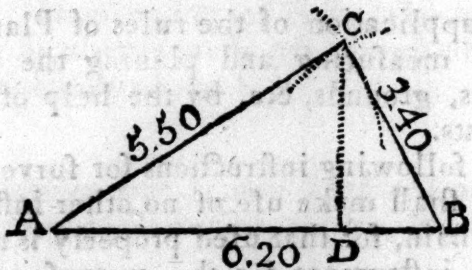
Problem I.

To measure a three sided field.

Rule. Measure round it, putting down the length of each side separately.

Exam-

EXAMPLE. Suppose a field in the following form be to be measured.



I measure the side A B and find it 6.20 (or 6 chains and 20 links,) the side B C and find it 3.40, the side C A and find it 5.50, and it is done.

To plot it.

Draw a blank line at random as the line AB, then from any scale of equal parts take 6.20 with your dividers or compasses, and lay it on that line from A to B and draw it with ink; then from the scale of equal parts take 5.50, set one foot in A and describe an arch, and take 3.40, set one foot in B and describe another arch to cut the former arch in C; draw the lines AC and BC and you have the true form of your field.

To find the content.

With your compasses take the nearest distance from C to the line A B, and measure that on the same scale that you used before, which we will suppose to be 3.03 for the perpendicular CD; then multiply 3.03 by half 6.20 and the product 9.3930 is the content required; which to reduce into acres, roods, and perches, you must

must point off 5 figures thus, .93930; multiply
4
by 4, point off 5 figures and ———
the left figure is roods 3.75720; multiply
40
by 40, and point off 5 fi- ———
gures, the left is perches 30.28800.

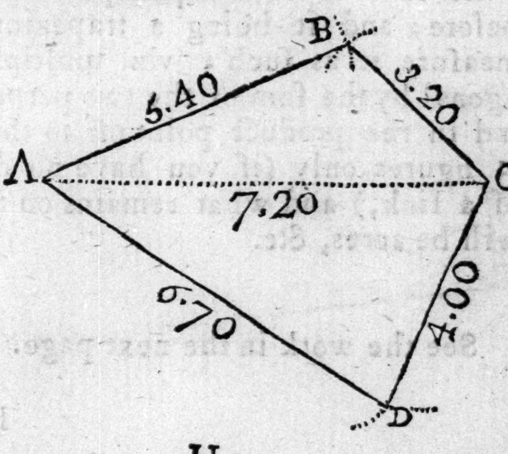
N. B. The reason of pointing off 5 figures to the right is the same as if the chain and links were set down and work'd as one whole number, and the product divided by 100000 (the square links in an acre;) for any number of chains is so many hundred links: Otherwise if you work the links as the decimal of a chain and divide the product by 10 (the square chains in an acre,) the answer in acres, &c. will be the same.

PROBLEM II.

To measure a four sided field.

RULE. Measure round it, putting down each side separately; and also measure one of the diagonals.

EXAMPLE. Suppose a field in the following form to be measured.



I meaſure the ſide A B and find it 5.40, the ſide B C and find it 3.20, the ſide C D 4.00, the ſide D A 6.70, and the diagonal A C 7.20, and its done.

To plot the ſame.

Draw the diagonal A C at random, then take 7.20 from any ſcale of equal parts and lay it from A to C; then take 5.40 in your compaſſes from the ſame ſcale, ſet one foot in A and deſcribe an arch; take 3.20 in your compaſſes from the ſame ſcale, ſet one foot in C and croſs the former arch in B; draw the lines A B and B C. Then take 6.70 in your compaſſes from your ſcale and deſcribe an arch, alſo take 4.00 in your compaſſes and croſs the laſt arch in D; draw the lines A D and D C, and ſo you have the true figure of the field.

To find the content.

Now let fall perpendiculars to the neareſt diſtance from the angles B and D upon the diagonal A C, and meaſure theſe perpendiculars by the ſame ſcale of equal parts that you uſed before; and it being a trapezium, you may meaſure it as ſuch; viz. multiply half the diagonal by the ſum of the two perpendiculars, and in the product point off to the right hand 5 figures only (if you have no decimal parts of a link,) and what remains on the left hand will be acres, &c.

See the work in the next page.

Perpen-

Perpendiculars. { 225 / 365   ½)720 = 360

```
         225
         365
        ----
         590
         360
       -----
       35400
       1770
     -------
     2.12400
           4
     -------
      .49600
          40
     -------
    19.84000
```

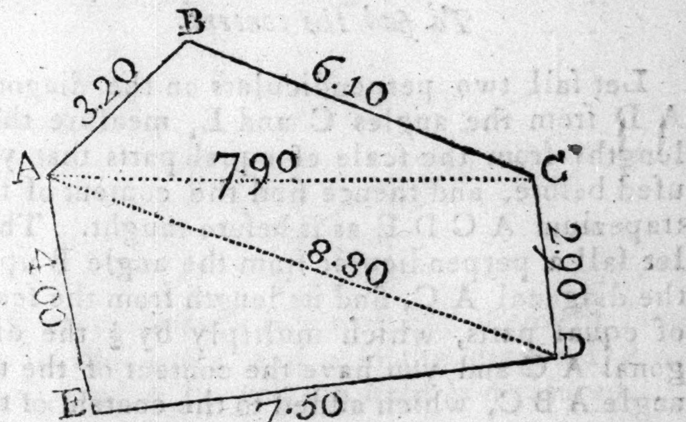
	A.	R.	P.
Content	2	0	19

PROBLEM III.

To measure a five sided field.

RULE. Measure round it, putting down each side separately; and also measure 2 diagonals.

EXAMPLE. Suppose a field in the following form to be measured.



I mea-

I meaſure the ſide A B and find it 3.20, B C and find it 6.10, C D 2.90, D E 7.50, and EA 4.00; the diagonal AC I find to be 7.90 and the diagonal A D to be 8.80.

To plot it out.

Draw the diagonal A C at random, and from a ſcale of equal parts take 7.90 and lay it from A to C; take 8.80 from the ſame ſcale, ſet one foot in A, and deſcribe an arch; take 2.90 from the ſcale, ſet one foot in C and croſs the former arch in D; draw the line A D for the ſecond diagonal, and the line C D for one ſide of the field. Then take 3.20 in your compaſſes and deſcribe an arch, and 6.10 and croſs that arch in B; draw he lines A B and B C; take 7.50 in your compaſſes and deſcribe an arch, alſo take 4.00 and croſs that arch in E; draw the lines DE and AE and you have the true figure of your field.

To find the content.

Let fall two perpendiculars on the diagonal A D from the angles C and E, meaſure their lengths from the ſcale ef equal parts that you uſed before, and thence find the content of the trapezium A C D E as is before taught. Then let fall a perpendicular from the angle B upon the diagonal A C, find its length from the ſcale of equal parts, which multiply by $\frac{1}{2}$ the diagonal A C and you have the content of the triangle A B C, which added to the content of the trapezium A C D E (before found) will give the

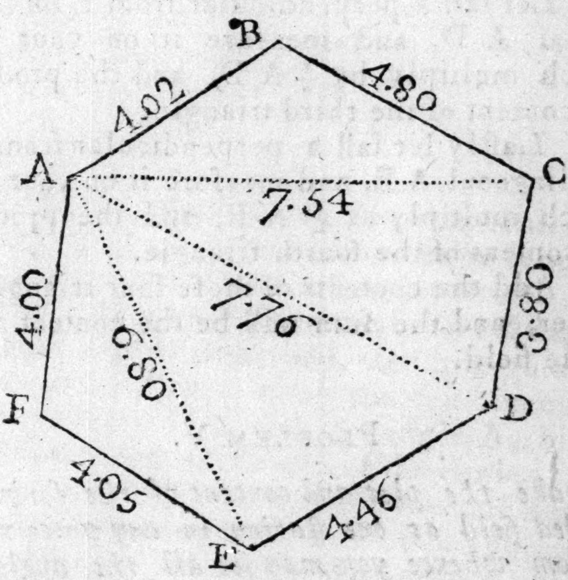
the content of the whole field as required; remembring to cut off for acres, &c. as before taught.

PROBLEM IV.

To measure a six sided field.

RULE. Measure round it, and put down each side separately; and also measure 3 of its diagonals.

EXAMPLE. Suppose a field in the following form to be measured.



I measure each side and find them the same as in the *figure*, and also the diagonal A C, A D, and A E, and the field is measured.

To plot the ſame proceed as in the laſt example; and for the area you have 4 triangles to find the content of.

1. Let fall a perpendicular from B to the diagonal A C, and meaſure it on your ſcale, which multiply by $\frac{1}{2}$ A C and you have the content of that triangle.

2. Let fall a perpendicular from C to the diagonal A D, and meaſure it on your ſcale, which multiply by $\frac{1}{2}$ AD, and the product is the content of the ſecond triangle.

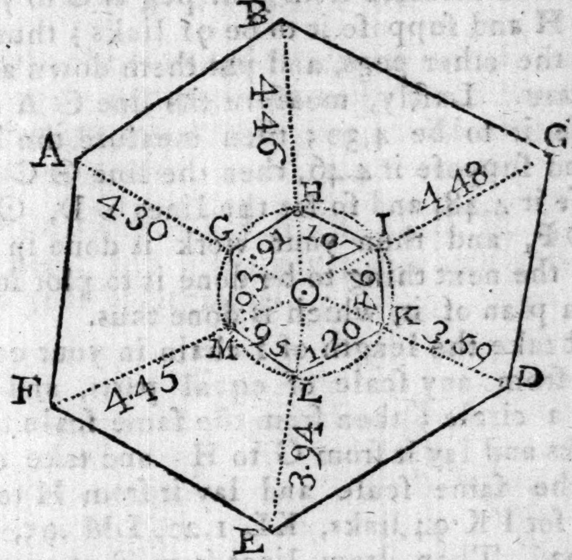
3. Let fall a perpendicular from E on the diagonal A D, and meaſure it on your ſcale, which multiply by $\frac{1}{2}$ A D, and the product is the content of the third triangle.

4. Laſtly let fall a perpendicular from F to the diagonal A E, and meaſure it on your ſcale, which multiply by $\frac{1}{2}$ A E, and the product is the content of the fourth triangle.

5. Add the contents of theſe four triangles together, and the ſum will be the content of the whole field.

PROBLEM V.

To take the plot and content of the former ſix ſided field at one ſtation in any place thereof from whence you may ſee all the angles, as in the following example.



Going into the field choose ſome convenient place therein, as at ⊙, and let marks be ſet up at every angle; then take hold of one end of the chain and ſtand in ⊙, and cauſe an aſſiſtant to take hold of the other end, let him go towards the angle A, and when you ſee him exactly in a ſtraight line with the angle A, bid him put down a peg at the end of the chain and leave it there at G; then cauſe your aſſiſtant to go towards the angle B, and when you ſee him in a ſtraight line with the angle B, bid him put down another peg at the end of the chain and leave it there at H; then cauſe him to go towards the angle C, and put down a peg at the end of the chain at I; thus do for all the other angles. And when you are come to M and put down your laſt peg, meaſure from it, to your firſt peg at G, which we will ſuppoſe 93 links;

links; then measure from your peg at G to your peg at H and suppose it to be 91 links; thus do for all the other pegs, and put them down as in the *figure*. Lastly, measure the line ⊙ A and suppose it to be 4.30; then measure the line ⊙ B and suppose it 4.46, then the line ⊙ C and suppose it 4.48, and so for the lines ⊙ D, ⊙ E, and ⊙ F, and then your work is done in the field; the next thing to be done is to plot it, or make a plan of it, which is done thus.

First take the length of 1 chain in your compasses from any scale of equal parts, and describe a circle; then from the same scale take 91 links and lay it from G to H; and take 1.07 from the same scale and lay it from H to I; and so for I K 94 links, KL 1.20, LM .95, and MG .93. Then draw lines from ⊙ at random through these several points G, H, I, K, L, M; and from the same scale of equal parts take 4.30 in your compasses and lay it from ⊙ to A; then take 4.46 and lay it from ⊙ to B; do the same for the other lines ⊙ C, ⊙ D, ⊙ E, ⊙ F; then draw the boundary lines A B, B C, D E, E F, F A, and thus you have the true figure of your field.

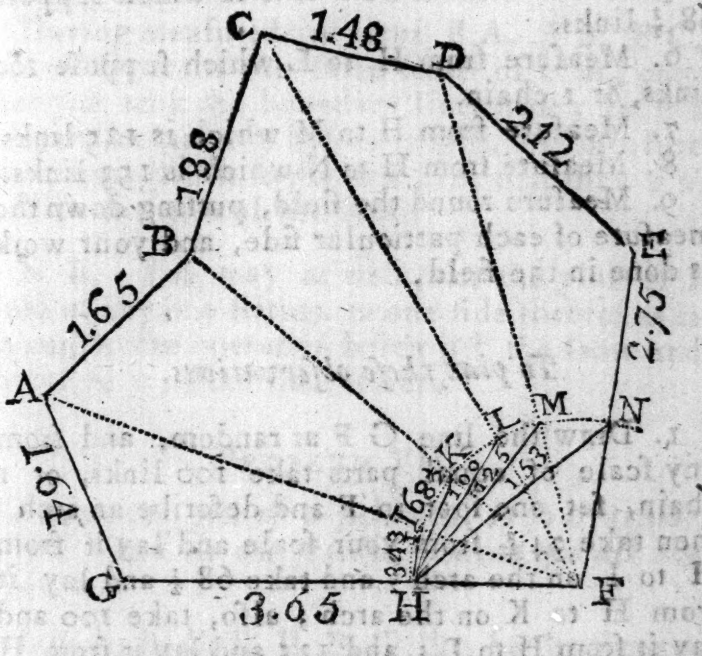
To find the content of this survey, observe the rules already laid down; viz. find the content of each single triangle (being here 6) and add their contents together into one sum, and that will be the content of the whole field.

PRO-

PROBLEM VI.

To take the plan and content of a field at one ſtation in any angle thereof from whence the other angles may be ſeen, without meaſuring the diagonals.

EXAMPLE. Let A, B, C, D, E, F, G, be the field, and F the angle at which you would take your obſervations.



1. Let an aſſiſtant take the chain and go towards the angle G while you ſtand with the other end of the chain in the angle F; bid him put down a peg at the end of the chain at H, and leave it there.

2. Then

2. Then cauſe him to go towards the angle A, and when you ſee him in a ſtraight line with A, bid him put down another peg at I, and leave it there alſo.

3. Cauſe him to go towards the angle B, and when you ſee him in a ſtraight line with B, bid him put down a peg at the end of the chain at K; proceed in this manner for the angles C, D, and E.

4. Then go to the peg at H and meaſure to the peg at I which ſuppoſe 34 $\frac{3}{4}$ links.

5. Then meaſure from H to K which ſuppoſe 68 $\frac{1}{2}$ links.

6. Meaſure from H to L which ſuppoſe 100 links, or 1 chain.

7. Meaſure from H to M which is 125 links.

8. Meaſure from H to N which is 153 links.

9. Meaſure round the field, putting down the meaſure of each particular ſide, and your work is done in the field.

To plan theſe obſervations.

1. Draw the line G F at random, and from any ſcale of equal parts take 100 links, or 1 chain, ſet one foot in F and deſcribe an arch; then take 34 $\frac{3}{4}$ from your ſcale and lay it from H to I on the arch; and take 68 $\frac{1}{2}$ and lay it from H to K on the arch; alſo, take 100 and lay it from H to L; and 125 and lay it from H to M; and take 153 and lay it from H to N; draw the lines F A, F B, F C, &c. thro' the points I, K, L, &c. at random.

2. From the ſame ſcale of equal parts take 3.65 and lay it from F to G; and take 1.64 in your

your compaſſes, ſet one foot in G and croſs the random line F A in A; alſo take 1.65, ſet one foot in A and croſs the random line F B in B; take 1.88 ſet one foot in B and croſs the random line F C in C; proceed in the ſame manner for all the reſt.

3. Draw the lines FG, GA, AB, BC, CD, DE, EF, and you have the true figure of the field.

To find the content.

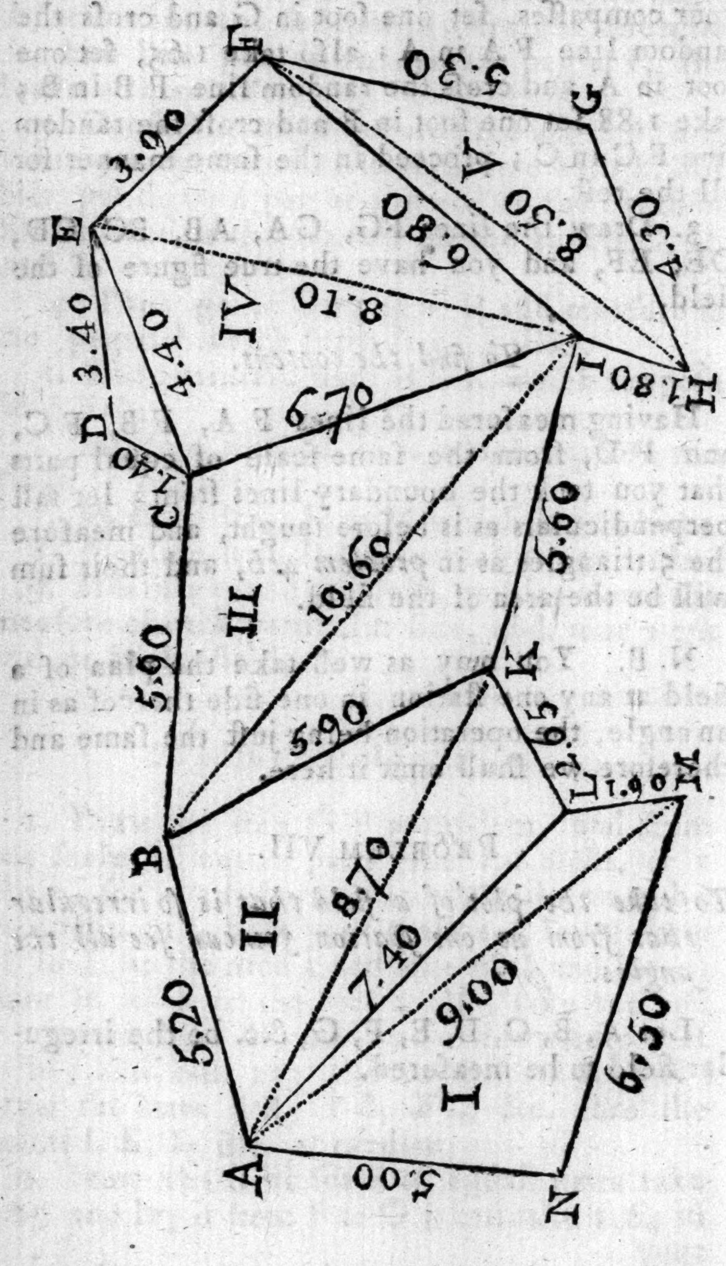
Having meaſured the lines F A, F B, F C, and F D, from the ſame ſcale of equal parts that you took the boundary lines from; let fall perpendiculars as is before taught, and meaſure the 5 triangles as in *problem 4th*, and their ſum will be the area of the field.

N. B. You may as well take the plan of a field at any one ſtation in one ſide thereof as in an angle, the operation being juſt the ſame and therefore we ſhall omit it here.

PROBLEM VII.

To take the plot of a field that is ſo irregular that from no one ſtation you can ſee all the angles.

Let A, B, C, D, E, F, G, &c. be the irregular field to be meaſured.


A
B
C
D
E
F
G
H
I
K
L
M
N
I
II
III
IV
V
5.00
5.20
5.90
1.40
3.40
3.90
5.30
4.30
1.80
5.60
2.65
1.90
6.50
9.00
7.40
8.70
5.90
10.60
6.70
4.40
8.10
6.80
8.50

I. Begin

1. Begin at A and meaſure the trapezium I as is taught in the former problems.

2. Meaſure the trapeziums II, III, IV, V, in like manner; and laſtly meaſure the triangle C D E.

3. Let fall perpendiculars from the angles which ſubtend the diagonals, upon the diagonals, and proceed for the areas of each ſingle trapezium, &c. as is before taught; the ſum of which will be the area of the field.

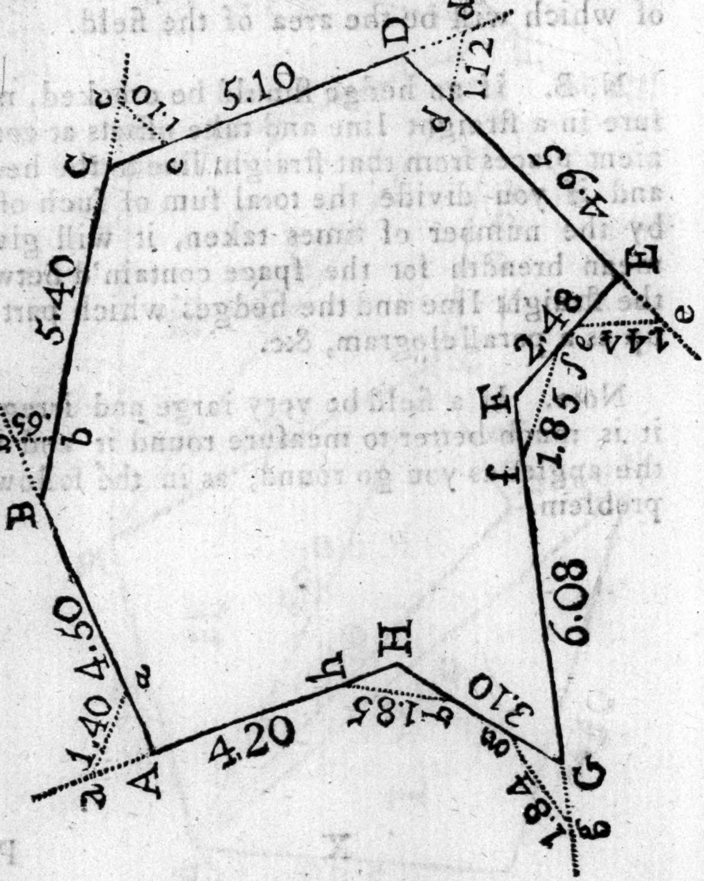
N. B. If an hedge ſhould be crooked, meaſure in a ſtraight line and take offsets at convenient places from that ſtraight line to the hedge; and if you divide the total ſum of ſuch offsets by the number of times taken, it will give a mean breadth for the ſpace contain'd between the ſtraight line and the hedge; which part caſt up as a parallelogram, &c.

Note. If a field be very large and irregular it is much better to meaſure round it and take the angles as you go round, as in the following problem.

PROBLEM VIII.

To take the plot of a wood, common, or a large champian field, by measuring round it, and making observations at every angle.

EXAMPLE. Let A, B, C, D, E, F, G, H, be a large field or wood through which you cannot see to take the angles, but must be forced to measure round it.



1. Begin

1. Begin at A and measure the line A B 4.50; when you are at B let an assistant take the chain straight forward (yourself standing at B,) bid him put down a peg at b and leave it there; then measure along the line B C one chain, put a peg at *b* and measure from b to *b* which suppose 65 links.

2. Measure the line B C 5.40; and when you are at C continue the chain straight forward and put down a peg at the end of it at c; measure from C to *c* one chain, put down another peg at the end of the chain, and measure the line c *c* which suppose 1.10.

3. Measure the line C D 5.10; and when you are at D measure one chain straight forwards and put a peg at d; measure from D to *d* one chain, put down a peg at the end of the chain, and measure the line d *d* which suppose 1.12.

4. Measure each particular side and angle in this manner till you have measured all round the field or wood, and then your work is done in the field; and what now remains to be done is to plot it and to find the content.

To plot it.

1. Draw the line A B at random, and from any scale of equal parts take 4.50 in your compasses and lay it from A to B; take 1 chain in your compasses, set one foot in B, and mark the line A B continued in b, draw an arch, and lay 65 from b on that arch to *b*; draw a line from B thro' the point *b* at random.

 2. Take

2. Take 5.40 in your compaſſes and lay from B to C; take 1 chain in your compaſſes, ſet one foot in C and mark the line BC continued in c, draw an arch, and lay 1.10 on that arch from c to *c*, draw a line from C through the point *c* at random.

3. Take 5.10 in your compaſſes and lay from C to D; take 1 chain in your compaſſes, ſet one foot in D and mark the line C D continued in d, draw an arch and lay 1.12 on that arch from d to *d*, draw a line from D thro' *d* at random.

4. Proceed juſt in the ſame manner for all the other lines and angles of the field; and if you have done your work right your laſt line H A will juſt come and join with the point A; which if it ſhould go beyond the point A you have laid off ſome of your angles too ſmall; but if it falls ſhort of the point A you have laid off ſome of your angles too large; and that error muſt be corrected accordingly.

To find the content.

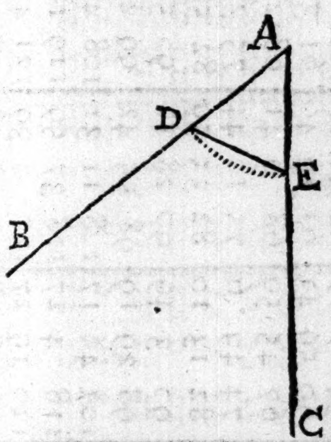
After you have drawn the plan juſtly and truly, you may divide it into trapeziums and triangles, and ſo let fall perpendiculars as is before taught, and thence find the content eaſily.

PRO-

PROBLEM IX.

To take any angle with the chain only.

EXAMPLE. Suppose the two sides of a field were as in the following figure; it is required to find the angle included between them?



First, I stand with one end of the chain in the angle A, and lay the chain along the hedge AB, and put down a peg at the end of the chain at D.

Secondly, I lay the chain along the hedge AC, and put down a peg at the end of the chain at E.

Thirdly, Measure from the peg at E to the peg at D, and suppose it measures 1.03; then look in the following table for 1.03 and you will find the angle to be 61 deg. 59 min. 42 sec. ——Or suppose the extent from D to E was 86 links; by the table you will find the angle to be 50 deg. 56 min. 6 sec.; and so for any other extent.

The following is a table of the degrees, minutes, and seconds, that is contained in any angle made by any number of links; radius being one chain, or 100 links.

	0			0 ½			1			1 ½			2			2 ½			3		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
	0	0	0	0	17	11	0	34	23	0	51	34	1	8	45	1	25	57	1	43	8
10	5	43	55	6	1	8	6	18	21	6	35	34	6	52	47	7	10	0	7	27	13
20	11	29	10	11	46	13	12	3	16	12	20	33	12	37	50	12	55	8	13	12	26
30	17	15	14	17	32	37	17	50	1	18	7	25	18	24	50	18	42	15	18	59	40
40	23	4	26	23	21	59	23	39	32	23	57	6	24	14	41	24	32	16	24	49	52
50	28	57	18	29	15	4	29	32	50	20	50	37	30	8	24	30	26	13	30	44	2
60	34	54	55	35	12	56	35	30	58	35	49	2	36	7	6	36	25	12	36	43	18
70	40	58	29	41	16	51	41	35	13	41	53	37	42	12	1	42	30	28	42	48	55
80	47	9	23	47	28	9	47	46	56	48	5	45	48	24	35	48	43	27	49	2	19
90	53	29	15	53	48	16	54	7	48	54	27	7	54	46	27	55	5	50	55	25	14
100	60	0	0	60	19	53	60	39	46	60	59	43	61	19	40	61	39	41	61	59	42
110	66	44	2	67	4	39	67	25	17	67	45	59	68	6	41	68	27	28	68	48	16
120	73	44	23	74	5	55	74	27	28	74	49	6	75	10	44	75	32	29	75	54	14
130	81	5	0	81	27	41	81	50	21	82	13	10	82	35	59	82	58	55	83	21	52
140	88	51	15	89	15	21	89	39	27	90	3	49	90	28	11	90	52	41	91	17	11
150	97	10	51	97	36	57	98	3	3	98	29	19	98	55	42	99	22	13	99	48	50
160	106	15	37	106	44	26	107	13	14	107	42	17	108	11	31	108	40	54	109	10	29
170	116	25	24	116	58	17	117	31	10	118	4	27	118	37	59	119	11	49	119	45	55
180	128	18	58	128	58	52	129	38	46	130	19	27	131	0	39	131	42	53	132	24	41
190	143	36	37	144	33	4	145	29	32	146	28	18	147	28	46	148	31	8	149	35	34

	3 ½			4			4 ½			5			5 ½			6			6 ½		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	2	0	20	2	17	31	2	34	42	2	51	54	3	9	6	3	26	18	3	42	30
10	7	44	27	8	1	41	8	18	55	8	36	9	8	53	23	9	10	38	9	27	52
20	13	29	47	13	47	9	14	4	25	14	21	41	14	39	1	14	56	21	15	13	41
30	19	17	6	19	34	33	19	51	59	20	9	26	20	26	54	20	44	22	21	1	51
40	25	7	29	25	25	6	25	42	48	26	0	21	26	18	1	26	35	41	26	53	20
50	31	1	52	31	19	42	31	37	34	31	55	26	32	13	17	32	31	9	32	49	6
60	37	1	25	37	19	33	37	37	43	37	55	52	38	14	3	38	32	15	38	50	28
70	43	7	24	43	25	52	43	44	23	44	2	55	44	21	29	44	40	3	44	58	39
80	49	21	14	49	40	9	49	59	7	50	18	5	50	37	5	50	56	5	51	15	10
90	55	44	43	56	4	12	56	23	40	56	43	7	57	2	40	57	22	14	57	41	52
100	62	19	47	62	39	52	63	0	2	63	20	11	63	40	25	64	0	39	64	20	57
110	69	9	9	69	30	2	69	50	59	70	11	57	70	33	1	70	54	4	71	15	12
120	76	16	5	76	37	56	76	59	54	77	21	52	77	43	56	78	6	1	78	28	12
130	83	44	58	84	8	3	84	31	19	84	54	36	85	17	55	85	41	13	86	4	45
140	91	41	52	92	6	32	92	31	24	92	56	16	93	21	19	93	46	22	94	11	37
150	100	15	35	100	42	28	101	9	28	101	36	36	102	3	52	102	31	16	102	58	22
160	109	40	13	110	10	30	110	40	18	111	10	36	111	41	8	112	11	51	112	42	46
170	120	20	20	120	55	2	121	30	3	122	5	24	122	41	4	123	17	5	123	53	27
180	133	7	36	133	51	8	134	35	19	135	20	12	136	5	48	136	52	11	137	39	21
190	150	42	18	151	51	37	153	3	50	154	19	22	155	39	43	157	2	35	158	31	50

	7			7 ½			8			8 ½			9			9 ½		
°	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	4	0	41	4	17	53	4	35	6	4	52	18	5	9	30	5	26	42
10	9	45	7	10	2	22	10	19	38	10	36	54	10	54	9	11	11	39
20	15	31	2	15	48	23	16	5	44	16	23	6	16	40	29	16	57	52
30	21	19	20	21	36	50	21	54	20	22	11	51	22	29	22	22	46	54
40	27	10	59	27	28	41	27	46	23	28	4	6	28	21	49	28	39	34
50	33	7	4	33	25	0	33	42	57	34	0	55	34	18	54	34	36	55
60	39	8	42	39	26	58	39	45	13	40	3	31	40	21	49	40	40	9
70	45	17	15	45	36	3	45	54	52	46	13	24	46	31	55	46	50	39
80	51	34	14	51	53	21	52	12	28	52	31	38	52	50	48	53	10	1
90	58	1	30	58	21	11	58	40	52	59	0	37	59	20	22	59	40	11
100	64	41	16	65	1	45	65	22	13	65	42	35	66	2	58	66	23	30
110	71	36	21	71	57	24	72	19	27	72	40	28	73	1	30	73	22	57
120	78	50	24	79	12	42	79	35	1	79	57	27	80	19	53	80	42	26
130	86	28	16	86	51	56	87	15	37	87	39	26	88	3	16	88	27	15
140	94	36	52	95	2	19	95	27	47	95	53	26	96	19	6	96	44	58
150	103	26	29	103	54	18	104	22	16	104	50	21	105	18	38	105	47	3
160	113	13	54	113	45	15	114	16	13	114	48	36	115	20	38	115	52	54
170	124	30	11	125	7	17	125	44	19	126	22	41	127	1	1	127	39	46
180	138	27	22	139	16	18	140	6	11	140	57	52	141	49	4	142	42	13
190	160	7	38	161	51	45	163	46	51	165	57	24	168	32	11	171	53	44

PROBLEM X.

To meaſure any inacceſſible diſtance by the chain.

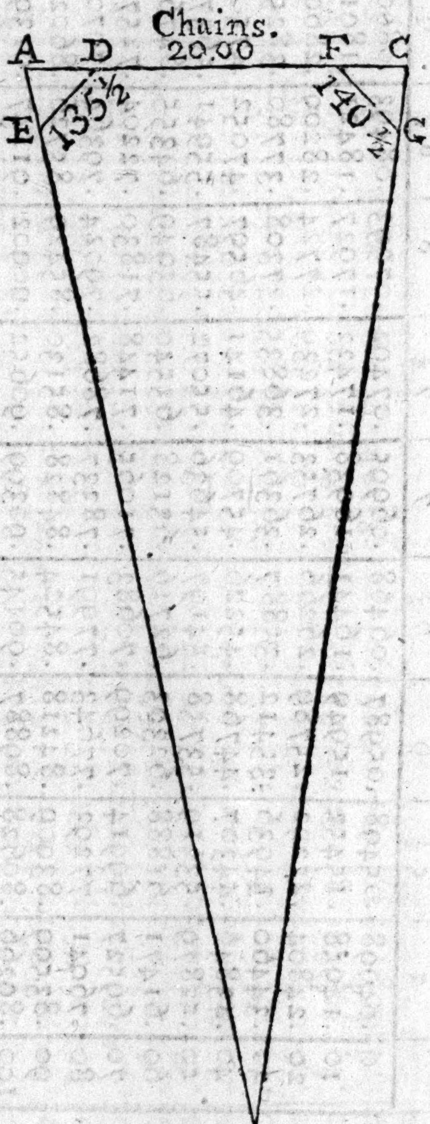
RULE I. Chooſe two ſtations, and meaſure the diſtance between them; then ſtand at your firſt ſtation, and lay off a chain towards your ſecond ſtation, and leave a peg at the end of the chain; then lay off a chain towards your object, and leave a peg at the end of your chain; meaſure your diſtance between your two pegs, and your work is done at your firſt ſtation. 2. Then meaſure to your ſecond ſtation, and lay one chain towards your firſt ſtation, and leave a peg at the end of the chain; then lay off a chain towards your object and leave a peg at the end of your chain; meaſure the diſtance between your two pegs, and your work is done at your ſecond ſtation. 3. Look in the 1ſt. table for the number of links that was between the pegs at each ſtation, add the two numbers together which you find againſt ſuch numbers in the table, and take the ſupplement of that ſum, that is, ſubtract the ſum from 180 degrees. Then look in the 2d. table which ſhew the natural ſines of the firſt, and ſay thus; As the ſine of the remainder is to the diſtance between the two ſtations, ſo is the ſine of the angle at the firſt ſtation to the diſtance of the object from the ſecond ſtation; and ſo is the ſine of the angle at the ſecond ſtation, to the diſtance of the object from the firſt ſtation. And thus you have two diſtances much more correct than if you had made uſe of the beſt theodolite or plain-table that ever was made.

Next follows a table of factors to find the area of any plain triangle, by having any two ſides and the angle included between them.

	0	0½	1	1½	2	2½	3	3½	4	4½
0	.00000	.00500	.01000	.01500	.01999	.02500	.02999	.03499	.03999	.04498
10	.09987	.10485	.10983	.11479	.11975	.12475	.12972	.13469	.13966	.14462
20	.19913	.20399	.20874	.21375	.21866	.22357	.22847	.23338	.23829	.24317
30	.29661	.30143	.30625	.31107	.31588	.32068	.32548	.33027	.33505	.33983
40	.39192	.39660	.40129	.40596	.41063	.41529	.41995	.42459	.42922	.43387
50	.48412	.48864	.49374	.49763	.50211	.50659	.51105	.51551	.51994	.52437
60	.57236	.57665	.58093	.58520	.58945	.59370	.59793	.60214	.60635	.61054
70	.65573	.65975	.66376	.66775	.67172	.67569	.67964	.68357	.68748	.69139
80	.73321	.73691	.74059	.74426	.74791	.75154	.75515	.75875	.76232	.76588
90	.80373	.80701	.81035	.81363	.81688	.82012	.82334	.82654	.82972	.83287
100	.86602	.86890	.87175	.87455	.87738	.88016	.88290	.88563	.88833	.89101
110	.91868	.92103	.92335	.92565	.92791	.93015	.93235	.93453	.93668	.93879
120	.96000	.96173	.96343	.96510	.96673	.96833	.96989	.97142	.97291	.97436
130	.98791	.98892	.98987	.99079	.99167	.99251	.99330	.99406	.99476	.99543
140	.99980	.99991	.99998	.99999	.99997	.99987	.99975	.99956	.99932	.99903
150	.99216	.99118	.99014	.98904	.98788	.98665	.98537	.98401	.98259	.98109
160	.96000	.95762	.95517	.95264	.95002	.94731	.94452	.94165	.93864	.93562
170	.89549	.89123	.88685	.88234	.87771	.87295	.86807	.86305	.85791	.85521
180	.78460	.77735	.77000	.76240	.75458	.74647	.73832	.72984	.72113	.71216
190	.59327	.57998	.56652	.55235	.53760	.52222	.50614	.48931	.47162	.45300

	5	5 ½	6	6 ½	7	7 ½	8	8 ½	9	9 ½
0	.04998	.05498	.05987	.06468	.06995	.07494	.07993	.08493	.08991	.09489
10	.14958	.15453	.15949	.16441	.16938	.17433	.17927	.18421	.18914	.19414
20	.24804	.25292	.25789	.26266	.26753	.27239	.27724	.28209	.28694	.29178
30	.34460	.34936	.35412	.35887	.36361	.36835	.37308	.37780	.38251	.38722
40	.43846	.44307	.44768	.45226	.45709	.46141	.46597	.47052	,47506	.47960
50	.52879	.53319	.53758	.54197	.54636	.55072	.55487	.55941	.56374	.56567
60	.61471	.61888	.62303	.62716	.63128	.63540	.63949	.64355	.64763	.65169
70	.69527	.69914	.70299	.70683	.71065	.71448	.71830	.72204	.72576	.72950
80	.76941	.77293	.77643	.77991	.78337	.78682	.79024	.79364	.79702	.80039
90	.83599	.83909	.84218	.84524	.84828	.85130	.85429	.85726	.86020	.86313
100	.89366	.89628	.89887	.90145	.90399	.90652	.90902	.91147	.91390	.91630
110	.94087	.94274	.94495	.94695	.94889	.95087	.95279	.95463	.95643	.95823
120	.97578	.97716	.97851	.97982	.98109	.98232	.98352	.98468	.98579	.98688
130	.99606	.99664	.99717	.99766	.99810	.99850	.99886	.99916	.99942	.99964
140	.99869	.99829	.99783	.99732	.99676	.99614	.99546	.99472	.99393	.99307
150	97954	.97791	.97621	.97448	.97261	.97069	.96871	.96665	.96451	.96229
160	.93247	.92923	.92589	.92245	.91892	.91528	.91162	.90770	.90375	.89970
170	.84721	.84166	.83595	83010	.82410	.81794	.81169	.80512	.79846	.79162
180	.70294	.69344	.68366	.67358	66329	.65247	.64141	.62980	.61816	.60594
190	.43330	.41212	.39004	.36600	.33998	.31130	.27932	.24266	.19875	.14098

EXAMPLE. Suppoſe I ſee a remarkable object B at a great diſtance, and I would know my diſtance from it.



1. Meaſure the diſtance A C which ſuppoſe 20 chains, lay off a chain from A to D, and alſo from A to E, meaſure the diſtance DE, (which ſuppoſe) 135 ½ links.

2. Go to your 2d. ſtation at C, lay off a chain from C to F, and alſo from C to G, meaſure the diſtance FG, (which ſuppoſe) 140 ½ links.

3. Look in the table for 135 ½ links and you will find it to be

	°	′	″
	85	17	55
look for 140 ½ & against it is	89	15	21
ſum	174	33	16
which ſubtracted from 180 deg. leaves	5	26	44

4. Look in the firſt table for 5 deg. 26 min. 44 ſec. and the neareſt to it is 5 deg. 26 min. 42 ſec. (which is near enough,) and the number of links belonging to it is 9 ½. Then look in the ſecond table for 9 ½, and the number belonging is .09489; look for 140½ and the number is .99991: Then ſay,

	Ch.		Ch.
As .09489 :	20.00 : :	.99991 : :	210.75,

which is 2 miles, 5 furlongs, and 16 ½ yards, for the diſtance AB.

Again,

	Ch.		Ch.
As .09489 :	20.00 : :	.99664 :	210.06

or 2 M. 5 F. 1 Y. 1 F. for the diſt. B C.

Note. If the number of degrees minutes, &c. in the ſupplement do not fall out very near to the ſame ſought in the table, and you would have your ſine more exact; ſay, As the difference between any two numbers in the firſt table is to the difference of the like ſines in the ſecond table, ſo is what is wanting of the firſt to a number to be added to the ſecond, &c.

PROBLEM XI.

To take a map or plan of any country or kingdom by the chain only.

1. Chooſe ſome remarkable hill or other place where you can ſee a number of towns, villages, pariſh churches, gentlemen's ſeats, mills, &c. and chooſe alſo a ſecond ſtation.

2. Lay off a chain from your place of ſtanding towards each particular object, remembering to leave the pegs ſtanding at each place; but lay one off towards your ſecond ſtation firſt.

3. Meaſure the exact diſtance from one peg to another even to tenths of a link, which write down in your rough ſketch, or elſe in your book; and your work is done at your firſt ſtation.

4. Find your diſtance to your ſecond ſtation by *problem 10th* (if the land between you and it is very unlevel and mountainous,) but if it is tolerable level and the diſtance not very great meaſure it with the chain.

5. Now you are at your ſecond ſtation lay off a chain from your place of ſtanding towards your firſt ſtation, and alſo towards each particular object juſt in the ſame manner and order that you did at your firſt ſtation; chooſe alſo a third ſtation and meaſure from peg to peg as you did before.

6. Find your diſtance to your third ſtation, and take the new objects which preſent themſelves at the ſecond and third ſtations juſt in the ſame manner as you did at the firſt and ſecond ſtations; proceed in this manner to a fourth, fifth, ſixth, &c. ſtations, till you have been all over the country or kingdom that you would take the plan of. But remember to chooſe your ſtations as far from one one another as you conveniently can, the farther they are from one another the better and more exact your work will be.

To plan your observations there are ſeveral methods, but we ſhall mention three, and ſo a perſon may chooſe which he will according as his work requires expedition or exactneſs.

I. METHOD.

Find the diſtance of each particular place or object from your two ſtations by *problem* 10*th*, then from any ſcale of equal parts take the numbers found, for the 1ſt. ſtation, ſet one foot in the 2d. ſtation and deſcribe an arch; take alſo the numbers found for the 2d. ſtation, ſet one foot in the 1ſt. ſtation and croſs the former arch, and that is the place of your object. Proceed in this manner for each of your other objects.

N. B. This is the moſt exact of any method, but it is too troubleſome for practice eſpecially when there are a great number of objects, becauſe two diſtances muſt be found before you can find the place of your object.

2d. METHOD.

Find the angles in the firſt table anſwering to your numbers found by obſervation; lay the center of the ſemicircle to your firſt ſtation and point off thoſe angles; draw lines at random from your firſt ſtation thro' each of theſe points; then do the ſame at your ſecond ſtation as you have done at your firſt, and the places where each correſponding line interſects is the place of that object belonging to thoſe lines.

N. B. This is a very good method, and much more expeditious than the former; tho' without great care you are liable to commit a ſmall error in pointing off the angles by the edge of the ſemicircle.

3d. ME-

3d. METHOD.

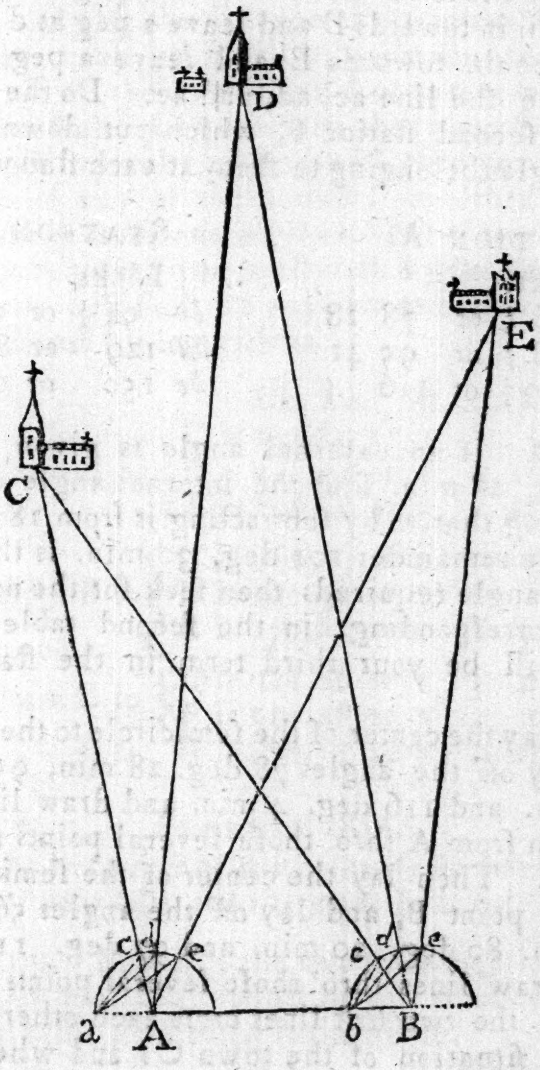
Having laid off the diſtance of your ſtations, take one chain in your compaſſes, ſet one foot in your firſt ſtation and deſcribe a circle, and alſo deſcribe a circle at your ſecond ſtation; then from your ſcale of equal parts take the very ſame numbers that you found by obſervation and lay them on the periphery of the circle, (which if right will end exactly at the very point that you ſet out from) draw lines from your ſtations thro' theſe reſpective points in the circle, and where the correſponding lines interſect, that will be the place of the particular object belonging thereto.

N. B. This is the moſt expeditious of the three methods, but in laying off the points on the periphery of the circle you will be very apt either to fall ſhort of, or go beyond the point that you ſet out from, and that error muſt be corrected by being more careful in pointing off till you come to the very point itſelf that you ſet out from.

To avoid running into this error, inſtead of deſcribing a circle with an extent of one chain only, take from your ſcale the diſtance of 2, 3, 4, 5, 6, &c. chains and deſcribe a circle, and lay on the periphery 2, 3, 4, 5, 6, &c. times the numbers found by obſervation; and if you have a good correct ſcale and ſharp pointed compaſſes, you may draw your plan or map to what degree of correctneſs you pleaſe.

EXAM-

EXAMPLE. Suppose I see three towns at an unknown distance, and I would know my distance from each of them, and also the distance of the towns from each other.



1, Choose two stations at a convenient distance from each other as A B 64 chains; then stand at the station A, and put down a peg at a in a straight line with the station B; lay off a chain towards the town C, and leave a peg at c; lay off a chain towards D and leave a peg at d; lay off a chain towards E and leave a peg at e; measure the line ac, ad, and ae. Do the same at the second station B, which put down with the angles belonging to them at each station.

Station A.					Station B.				
	Links.		°	′		Links.		°	′
ac	126 $\frac{1}{2}$	or	78	28	*bc*	94 $\frac{1}{2}$	or	56	24
ad	148 $\frac{1}{4}$	or	95	41	*bd*	129	or	80	20
ae	169 $\frac{2}{3}$	or	116	4	*be*	150	or	97	11

N. B. If an external angle is given as ac 78 deg. 28 min. find the internal angle corresponding thereto by subtracting it from 180 deg. and the remainder 101 deg. 30 min. is the internal angle required; then seek for the natural sine corresponding, in the second table, and that will be your third term in the stating.

2. Lay the center of the semicircle to the point A; lay off the angles 78 deg. 28 min, 95 deg. 41 min. and 116 deg. 4 min. and draw lines at random from A thro' those several points marked off. Then lay the center of the semicircle to the point B, and lay off the angles 56 deg. 24 min. 80 deg. 20 min. and 97 deg. 11 min. and draw lines thro' those several points; and where the two first lines cross each other, that is the situation of the town C; and where th

two

two second lines cross each other, that is the place of the town D; and where the two third lines cross each other, that is the place of the town E.

Now the distance of each town from your stations and also the distance of the towns from one another may be found by extending from your stations to the towns and that extent measured on that scale of equal parts that your stationary distance was taken from, will give you the distance of any of the towns from either of your stations or from one another. But if more exactness is required you must work by *problem 10th.* and you will find the answers as below, which were found by logarithms.

Thus the distance		Ch. lin.		M.	F.	Yds.
from A to	C is	141.89	or	1	6	41 ½
	D is	238.34	or	2	7	183 ½
	E is	196.20	or	2	3	136 $\frac{2}{5}$
from B to	C is	166.91	or	2	0	152
	D is	240.58	or	3	0	12 ¾
	E is	177.63	or	2	1	167 ¾
from C to	D is	111.06	or	1	3	23 ⅓
	E is	120.48	or	1	4	10 ½
D to	E is	87.34	or	1	0	161 ½

PROBLEM XII.

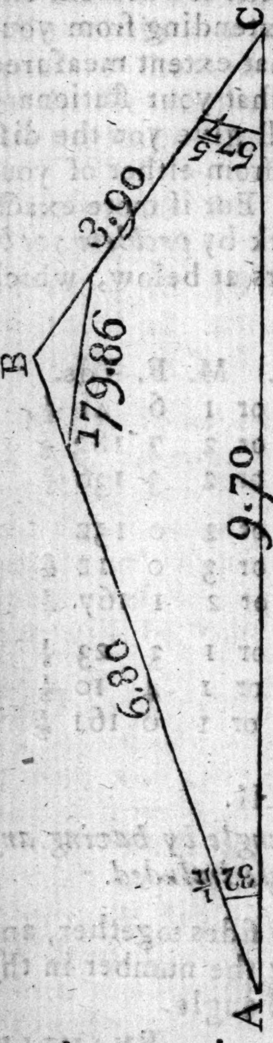
To find the content of any triangle by having any two sides and the angle included.

RULE. Multiply the two sides together, and multiply half the product by the number in the second table answering to the angle.

EXAMPLE.

Example. Suppose a triangle as in the following figure, what is its area.

N. B. In order to value the fraction of a link in an angle, multiply the difference of any two whole numbers in the table of sines by the fractional part of a link, and add the product to the lesser number till you come to $141\frac{1}{2}$ links; but subtract the value of the difference from the greater number (for the proper factor) if it is greater than $141\frac{1}{2}$ links.



AC	9.70
AB	6.80
	77600
	5820
	½)65.9600
	32.9800
	.31675
	1649000
	2308600
	1978800
	329800
	989400
Area	1.0446415000

See the operations of the different angles in the next page.

CA 9.70	BA 6.80
CB 3.90	BC 3.90
87300	61200
2910	2040
½)37.8300	½)26.5200
18.9150	13.2600
.55244	.78654
756600	530400
756600	663000
378300	795600
945750	1060800
945750	928200
1.0449402600	1.0429520400

In each product you'll observe one more decimal pointed off than what are in the factors, in order to produce acres, &c. And here we have worked the queſtion three ways, by the 3 different factors belonging to the 3 different angles, and each operation is very nearly the ſame; ſo that it is a matter of indifference which two ſides of a triangle are known (if the angle included is known alſo) to find the area of the triangle; and this is at any time done without having the plan or plot of the triangle.

Now let us find the content of the aforemention'd triangle by the old method when the plan is not given, and then we ſhall ſee how much more troubleſome it is than the new method.

See the work in the next page.

AC	9.70	10.20	10.20	10.20
AB	6.80	9.70	6.80	3.90
BC	3.90	.50	3.40	6.30

½)20.40

10.20
6.30

30600
6120

64.2600
3.40

25704000
1927800

218.484000
.50

109.24200000(1.04518
1

204)0924
816

2085)10820
10425

20901)39500
20901

209028)1859900
1672224

187676

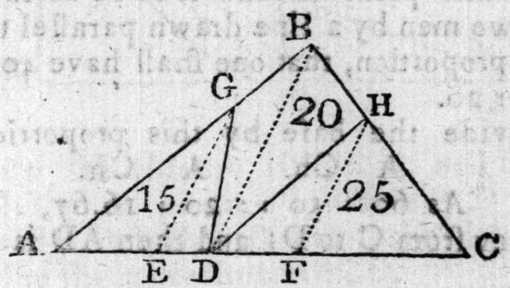
The

The Method of Surveying, *as it relates to* dividing of Land.

PROBLEM XIII.

To divide a triangular piece of land into any number of equal or unequal parts, by lines proceeding from any point aſſigned in any ſide thereof.

EXAMP. Let ABC be the triangular piece of land, containing 60 acres, to be divided between three men; the firſt to have 15 acres, the ſecond 20, and the third 25 acres; and the lines of diviſion to proceed from D.



Firſt meaſure the baſe which is 50 chains; then divide this baſe into 3 parts, thus;

	A		Ch.		A		Ch.	
As	60	:	50	::	15	:	12.50	for the

firſt man's baſe, which ſet from A to E; then ſay, As 60 : 50 :: 20 : 16.67 for the ſecond

ſecond man's baſe, which ſet from E to F; and the third man's baſe muſt be 20.83, viz. from F to C. This done draw an obſcure line from D to the oppoſite angle B, and from E and F draw the lines E G and F H parallel to B D. Laſtly from D draw D G and D H, which will divide the triangle into three ſuch parts as were required.

Problem XIV.

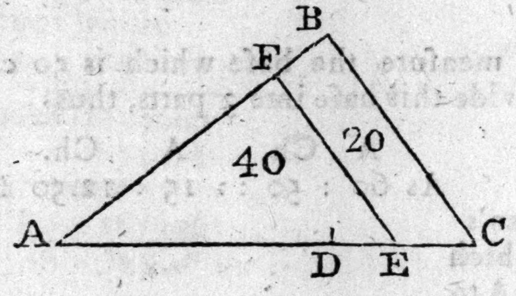
To divide a triangular piece of land according to any proportion given by a line drawn parallel to one of its ſides.

Examp. Let ABC be the triangular piece of land containing 60 acres, the baſe A C is 50 chains; this piece of land is to be divided between two men by a line drawn parallel to BC, in ſuch proportion, that one ſhall have 40 acres, the other 20.

1. Divide the baſe by this proportion,

A Ch. A Ch.
As 60 : 50 : : 20 : 16.67,

which lay from C to D; and then AD is 33 33.



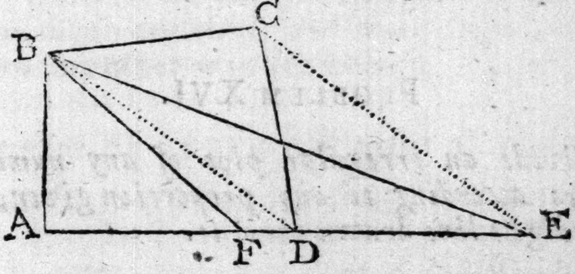
2. Find

2. Find a mean proportional between A D and A C, viz. 40.82, which set from A to E; then draw EF parallel to BC, and the triangle is divided as required.

PROBLEM XV.

To reduce a trapezium into a triangle, by lines drawn from any angle thereof.

Let A B C D be the trapezium to be reduced into a triangle and B the angle assigned.



Draw the obscure line B D, and draw C E parallel to it, produce the base A D to E, and draw BE, which will make the triangle ABE equal to the trapezium ABCD.

Now to divide this trapezium according to any assigned proportion, is no more than to divide the triangle ABE, as is before taught in *problem* 13*th.* which will also divide the trapezium.

EXAMPLE.

Suppose the trapezium A B C D contains 124 acres, 3 roods, 8 perches, is to be divided

Z between

between two men; the first to have 50 Ac. 2 R. 3 P. and the other 74 Ac. 1 R. 5 P. and the line of division to proceed from B.

1. Reduce the shares into perches, and it will be 8083 for the first man, and 11885 for the second man's share.

2. Measure the base of the triangle, viz. AE 78 chains; then say,

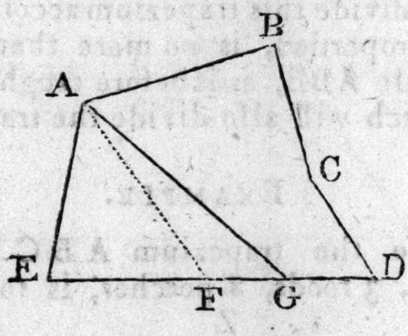
	Per.	Ch.	Per.	Ch. l.
As	19968 :	78 : :	8083 :	31.52,

which set off from A to F, draw the line BF and the trapezium is divided as required; the triangle ABF being the first man's share, and the trapezium BCDF the second's.

Problem XVI.

To divide an irregular plot of any number of sides according to any proportion given, by a straight line drawn thro' it.

Suppose the field ABCDE contains 46 acres to be divided into two equal parts between two men, by a line proceeding from A.



1. Draw

1. Draw a line at pleasure thro' the *figure*, as AF; then cast up the content of either half, and see what it wants, or what it is more than the true half should be.

2. Thus I cast up the content of AEF, and find it to be but 15 acres; whereas the true half is 23 acres; 8 acres being in the part ABCDF more than in AEF; therefore I make a triangle containing 8 acres, and add it to A E F, as the triangle A G F; then the line A G parts the *figure* into two equal parts.

Thus you may divide any piece of land of never so many sides and angles, according to any proportion, by straight lines drawn thro' it, with as much certainty, and more expeditiously than by any other way yet known.

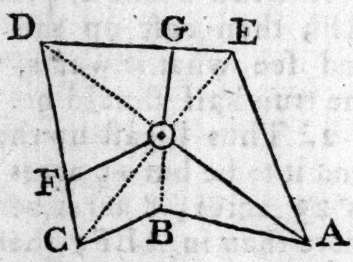
Another EXAMPLE will make all plain.

Suppose the following field, containing 27 acres, is to be divided between 3 men, each to have 9 acres, and the lines of division to run from a pond in the field so that each may have the benefit of the water, without going over one another's land.

1. From the pond ⊙ draw lines to every angle, as ⊙A, ⊙B, ⊙C, ⊙D, ⊙E; and then the *figure* is divided into 5 triangles, each of which measure, and put down the contents severally; which contents reduced into perches will stand thus,

A⊙B = 674
B⊙C = 390
C⊙D = 1238
D⊙E = 911
E⊙A = 1107 } Perches.

4320



The whole content being 4320 perches or 27 acres, each man's proportion being 1440 perches.

From ⊙ to any angle draw a line for the division line, as ⊙A; then consider that the first triangle A ⊙ B is but 674 perches, and the second triangle B ⊙ C 390, both together make but 1064 perches, which is less than 1440 by 376. You must therefore cut off from the third angle C⊙D 376 perches for the first man's dividing line, which you must find out thus: The base DC is 18 chains, the content of the triangle 1238; then say,

P. C. P. C.
As 1238 : 18 : : 376 : 5.45

which set from C to F, and drawing ⊙F, you have the first man's part, viz. A⊙F.

Then see what remains of the triangle ⊙D, 376 being taken out you will find it to be 862 which is less than 1440 by 578: therefore from the triangle D⊙E cut off 578 perches, and the point of division will fall in G; draw the line ⊙G, which with ⊙A and ⊙F divides the *figure* into three equal parts.

PROBLEM XVII.

To find what point of the compass any hedge or other object is upon, by the chain.

RULE. 1. Lay off one chain along the hedge (leaving a peg there) and another straight towards the sun; take the distance between the two pegs, which look for in the first table, and take the angle belonging thereto. 2. Take the sun's altitude at that time with a quadrant, and also the hour and minute of the day, which being reduced into degrees and minutes of the equator, say thus,

As sine of the hour from noon is to the complement of the sun's altitude, so is the co-sine of the sun's declination to the sine of the sun's azimuth, which subtracted from 180 degrees gives the sun's azimuth from the south; and the former angle between the sun and hedge being added or subtracted (as occasion shall require) will give the true point of the compass that the hedge is upon.

QUEST. Suppose on April 10, at 18 minutes past 9 in the morning I observe the sun to be 33 deg. 50 min. high, and the angle between the hedge and sun to be 42 links, which is 24 deg. 14 min. 41 sec. and the hedge lies more remote from the south than the sun; I demand what point of the compass it is upon?

Here we have given the hour from noon 40 deg. 30 min. sun's altitude, 33 deg. 50 min. sun's declination north 8 deg. 3 min.

		°	′
As co-ſine * of the ſun's altitude		56	10
To ſine of the hour from noon	-	40	30
So is co-ſine of the ſun's declination		81	57
To ſine of ſun's azimuth from ſouth		50	44

To which add the angle which belongs to 42 links, and the ſum is 74 deg. 58 min. 41 ſec. from the ſouth toward the eaſt, viz. ESE ¾ E fere.

N. B. The ſun's azimuth may be found by various other methods, which the ſurveyor may uſe as he ſees proper.

---

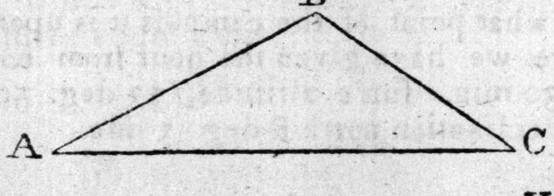
* Co-ſine or ſine complement is what the angle wants of 90 degrees.

Problem XVIII.

To take the horizontal line of an hill.

When you meaſure an hill you muſt meaſure the ſuperficies thereof, and accordingly caſt up the content; but when you plot it down, (becauſe you cannot make a convex ſuperficies on the paper) you muſt only plot the horizontal line or baſe thereof, which you muſt ſhadow over with the reſemblance of an hill. That horizontal line is found in this manner.

Suppoſe A B C be an hill whoſe baſe you would know.



Having

Having a quadrant with you, ſtand at A, cauſe a mark to be ſet up at B, ſo high above the top of the hill, as you hold the quadrant from the ground at A; then take the angle A 28 deg. 30 min. meaſure the diſtance AB 24 chains; then go to C and take the angle C 39 degrees, ſubtract the ſum of theſe two angles from 180 degrees and there remains 112 deg. 30 min. for the angle B; then ſay,

As ſine C : AB : : ſine B : AC; that is

As ſine 39° : 24 Ch. : : ſine $112^{\circ}\ 30'$: 35 23 Ch. l. AC.

N. B. The above may be done by the chain only, but it is a very prolix method; therefore we need not abuſe the chain by aſcribing methods thereto which may be eaſier attained by other means; and if there be any greater improvement capable of being made by the chain than what I have pointed out, I ſhall be more than amply ſatisfied if I have contributed any thing towards it.

OF

OF SPECIFIC GRAVITY.

SPecific gravity is the comparative weight of bodies of equal bulk. Thus if a cubic inch of fine copper be 9 times heavier than a cubic inch of water, we ſay the ſpecific gravity of copper is 9, or its weight is 9 times greater than that of water.

The ſpecific gravity of fluids are readily determined by weighing one and the ſame ſolid in them ſeverally; for ſince we ſuppoſe the ballance in equilibrio with the body ſuſpended in the air, the equilibrium muſt be deſtroyed when the ſolid is immerſed in the fluid, and muſt be then reſtored by weights put into that ſcale to which the body is appended; theſe weights will expreſs the gravity of an equal bulk of the reſpective fluids, and conſequently may be thus compared with each other, or all of them with the gravity of common water as uſual, and diſpoſed in a proper table, making that of water 1000.

In the ſame manner, if divers ſolids are firſt weighed in air, and then immerſed in the ſame fluid, as water, for inſtance, the equilibrium will be deſtroyed, which will be reſtored as before by putting in ſo much weight as is equal to the weight of the ſame bulk of water: The gravity therefore of every ſolid is thus compared with water, and conſequently with each other as in the following table.

Note. The application of this doctrine of ſpecific gravity is not only to compare the weight of a magnitude of one kind, in relation to another

ther kind of body, tho' this is of great use in computing the weights of such bodies as are too heavy or too unwieldly to have their weights discovered by other means, but 'tis of the utmost use in discovering the goodness of any kind of fossils, drugs, metals, precious stones, &c. whether they are genuine or counterfeits; hence appears the usefulness it is of to physicians, chemists, apothecaries, jewellers, goldsmiths, farmers, &c. the exact knowledge of which must conduce to their interest. But the great benefit it will be of to the several uses of life in making these matters familiar will be evident from the following instances: Having first premised that a cubic foot of common water weighs very exactly 1000 ounces avoirdupois weight, or 62 pounds and an half; which may be very readily reduced to troy weight, by a consideration of the rules laid down at the end of reduction.

A Table of metals and other bodies, shewing their specific gravities to rain water; as also the number of ounces in a solid foot, avoirdupois weight.

Bodies.	*Spec. grav.*	*Bodies.*	*Spec. grav.*
Air - -	1.2	Bees-wax, white	865
Ale - -	1028	Ditto, yellow	960
Aqua fortis -	1300	Bismoth -	9700
Aqua regia -	1234	Blue-slate -	3500
Ash, dry -	838	Blue-stone -	2740
Barley last year	658	Box, dry -	1030
Beech, dry -	700	Brass, cast -	8104
Bees-wax -	995	Brandy - -	927

Brazil

Bodies.	*Spec. grav.*
Brazil	1031
Brick	2000
Brimſtone	1800
Butter	940
Cedar	613
Chalk	2370
Clay	1718
Coal	1255
Copper	8795
Ditto, fine	9000
Ditto, oar	5775
Cork	240
Crabtree, dry	700
Diamond	3517
Earth	1984
Ebony	1177
Elm, new	800
Ditto, dry	600
Fir, yellow, dry	657
Do. white, dry	569
Flint-ſtone	2582
Free-ſtone	2352
Glaſs	2600
Gold, caſt, or ſtandard	18888
Do. fine, pure	19640
Horn	1832
Ice	908
Iron, caſt	7135
Ditto, bar	7764
Iron-ſtone	3600
Ivory	1832
Lead, fine	11313
Ditto, common	10130

Bodies.	*Spec. grav.*
Lead, oar	6800
Ditto, worſe	6200
Lignum vitæ	1327
Linſeed oil	923
Logwood	913
Magnet or Loadſtone	5106
Mahogany	1063
Man's body	1111
Marble	2707
Milk	1030
Oak ſappy, 1 year old	870
Heart of do. dry	925
Paving-ſtone	2460
Pitch	1190
Quickſilver	13762
Rain water	1000
River water	1009
Sea water	1030
Silver, caſt	10528
Ditto, fine	11091
Do. Engl. coin	10629
Spelter	7065
Spts. of Turpen.	874
Steel	7850
Sulpher, com. vide Brimſt.	1800
Sulpher-vivum	2000
Tin	7550
Ditto, oar	480
Wheat	757
Ditto meal	495
Yew	760

N. B. If the ſpecific gravity of any ſolid in the table be leſs than 1000 it will ſwim in water, but if greater than 1000 it will ſink.

As water is made the ſtandard for ſpecific gravity, its gravity is repreſented by unity or 1, or 1000 where three cyphers are added to give room to expreſs the gravities of other bodies in larger numbers in the table; in doing this we have a twofold advantage, the firſt is, that by this means we can expreſs the ſpecific gravities of bodies to a much greater degree of accuracy and exactneſs; the ſecond is, that the numbers in the table do alſo expreſs the avoirdupois ounces contained in a cubic foot of every ſort of matter therein ſpecified, becauſe a cubic foot of common water is found by experiment to weigh very nicely 1000 ounces.

Suppoſe I would know how many times any quantity of glaſs is heavier than the ſame quantity of water; then I find in the table againſt glaſs 2600, which if the point of unity was put would be 2.600, the meaning of which is that glaſs is $2\frac{6}{10}$ times heavier than the ſame quantity of water.

Now in order to find what relation a ſolid foot or 1000 ounces has to a ſolid inch, let us conſider that 1728 ſolid inches is 1000 ounces, and conſequently,

In.	oz.		In.
As 1728 :	1000 ::	1 :	.5787

of an ounce; ſo that a ſolid foot of water is to a ſolid inch of water as 1000 to .5787, which divided by 16 gives .036169 for the weight of a ſolid inch of water in the decimal parts of a pound.

Alſo

Alſo when you have gold, ſilver, electuaries, &c. which is generally weighed by troy weight, you muſt firſt reduce it into avoirdupois weight to agree with the foregoing table.

CASE I. *The dimenſions or ſolidity being given, to find the weight.*

RULE. Multiply the ſolidity in inches by the ſpecific gravity, and divide the product by 1728 (the ſolid inches in a foot,) and the quotient will be the weight in ounces avoirdupois.

If an oak beam be 120 inches long, and 11 by 9, how heavy is it?

$$
\begin{array}{r}
11 \\
9 \\
\hline
99 \\
120 \\
\hline
1980 \\
99 \\
\hline
11880 \\
925 \\
\hline
59400 \\
23760 \\
106920 \\
\hline
10989000
\end{array}
$$

See the remainder of the work in the next page.

1728)

```
                    Ounces.       lb.  oz. dra.
1728)10989000(6359.37  or  397   7   6
     10368      Answer.
     -----
      6210
      5184
      -----
      10260
       8640
      -----
      16200
      15552
      -----
        6480
        5184
       -----
       12960
       12096
       -----
         864
```

Note. As the ſpecific gravity of dry heart of oak is different from that which is ſappy, when you meet with oak that is part heart and part ſap, you muſt take ſuch proportion between theſe two numbers that there is between the heart and ſappy part; obſerve the ſame alſo with regard to the moiſture or dryneſs thereof.

Suppose an iron ball be 8 inches in diameter, what is its weight?

```
                8
                8
              ----
               64
                8
              ----
              512
            .5236
           ------
             3072
            1536
           1024
          2560
          --------
          268.0832
              7135
          --------
          13404160
         8042496
        2680832
       18765824
       -----------      Ounces.     lb.  oz.  dra.
1728)1912773.6320(1106.93     or    69    2    14
     1728                 Answer.
     ----
      1847
      1728
      -----
       11973
       10368
       -----
        16056
        15552
        -----
          5043
          5184
          ----
```

Suppofe I have a large grinding ftone for a blade mill, whofe diameter is 70 inches, and thicknefs 15 inches; what will the carriage come to at 3 d. per hundred weight?

```
           70
           70
        -----
         4900
        .7854
       ------
        19600
       24500
      39200
     34300
    -----------
     3848.4600
            15
    -----------
     192423000
     38484600
    -----------
     57726.9000
     2352
    -----------
    1154538
   2886345
  1731807
 1154538
 --------------
 135773668.8
```

See the remainder of the work in the two following pages.

```
                Ounces.
1728)135773668.8(78572.72
     12096
     -----                      lb.
      14813              16(78572.72(4910.795
      13824                 64
      -----                 --
       9896                 145
       8640                 144
       ----                 ---
       12566                 17
       12096                 16
       -----                 --
         4708                127
         3456                112
         ----                ---
         12528                152
         12096                144
         -----                ---
           4320                80
           3456                80
           ----                --
            864
```

See the remainder of the work in the next page.

```
   lb.   d.    lb.
As 112 : 3 : : 4910.795
                      3
           ------------  Pence      s.  d.
    112)14732.385(131.539 or 10 11½.156
        112                Answer.
        ----
         353
         336
         ----
          172
          112
          -------
           603
           560
           ----
            438
            336
            -----
            1025
            1008
            -----
              17
```

This is the method for bodies that are solid throughout; but for hollow bodies, one method is, to find the solidity of its greatest bulk and subtract the concavity therefrom, and proceed with the remainder as in the last example; but the following will make all clear, being done a nearer way.

If an iron bomb shell be 3 inches thick, and greatest diameter 14 inches, what is its weight?

14 greatest diameter.
6 twice the thickness.
8 least diameter.
8
64
8
512

14
14
56
14
196
14
784
196
2744
512
2232
.5236
13392
6696
4464
11160
1168.6752
7135
58433760
35060256
11686752
81807264
8338497.5520

See the remainder of the work in the next page.

```
         Ounces.       lb. oz.
1728)8338497(4825  or  301  9
     6912      Answer.
     -----
     14264
     13824
     -----
       4409
       3456
       ----
        9537
        8640
        ----
         897
```

CASE II. *The weight being given, to find the solid content.*

RULE. Divide the weight in ounces by the specific gravity, the quotient multiplied by 1728 will be the solidity in inches.

Suppose an iron shot weighs 49 lb. how many solid inches doth it contain? And also what is its diameter?

See the work in the next page.

49
16
294
49
7135)784.0(.10988
7135
70500
64215
62850
57080
57700
57080
620

.10988
1728
87904
21976
76916
10988
189.87264
Solid inches.

.5236)189.87264(362.629
15708
32792
31416
13766
10472
32944
31416
15280
10472
48080
47124
956

. . Inches.
362.629(7.13
343 Diameter.
14911)19629
14911
1518699)4718000
4556097
161903

How

How may ſolid inches are there in a marble ſtatue that is 280lb.?

```
       280
        16
      ----
      1680
      280
     -----
2707)4480(1.655
     2707
     -----
      17730
      16242
      -----
       14880
       13535
       -----
        13450
        13535
```

```
      1.655
       1728
     ------
      13240
      3310
     11585
     1655
   --------
Anſ. 2859.840
```

Note. In like manner the ſolid inches in a cup, tankard, glaſs bottle, &c. may be found, by making uſe of the proper ſpecific gravities, and working as above.

What must be the thickness of that lead, that one square foot may weigh 6 lb.?

6
16

10130)96,000(.009476
91170

48300
40520

77800
70910

68900
60780

8120

.009476
1728

75808
18952
66332
9476

144)16.374528(.11371 of an inch. Ans.
144

197
144

534
432

1025
1008

172

And in this manner you may find the thickness of any lead pipe, or other thing made of different materials, whose weight and superficial content are known; for if they are hollow cylindrical bodies, you must multiply the circumference by the length and that gives the superficies, and work with that as in the above example.

How

How many solid inches are there in a man's body that weighs 150 lb. ?

```
  150
   16
 ----
  900
 150
 ----
1111)2400(2.16
     2222
     ----
      1780
      1111
      ----
       6690
       6666
       ----
         24
```

```
  1728
  2.16
 -----
 10368
 1728
3456
-------
3732.48
Answer.
```

How many solid inches are there in a silver tankard that weighs 23 ounces (avoirdupois)?

```
10528)23.000(.002184
       21056
       -----
        19440
        10528
        -----
         89120
         84224
         -----
          48960
          42112
          -----
           6848
```

```
      .002184
         1728
      -------
        17472
        4368
      15288
      2184
     --------
Ans. 3.773952
```

CASE

CASE III. *The weight and magnitude being given, to find the ſpecific gravity.*

RULE. Divide the weight in ounces by the ſolidity in cubic feet, the quotient will be the ſpecific gravity.

If a piece of marble contains 4 ſolid feet, and weighs 675lb. what is its ſpecific gravity?

```
       675
        16
      ----
      4050
      675
     -----
   4)19800
     -----
Anſ.  2700
     -----
```

I have a piece of timber that contains 6 feet, and weighs 285lb. what wood is it?

```
      285
       16
     ----
     1710
     285
     ----
   6)4560
     ----
```

.760 ſpecific gravity; then look in the table of ſpecific gravities for that number, and the name of the wood againſt it is yew. But you muſt obſerve that the ſpecific gravity of any ſort of wood varies in the ſeaſoning; and other commodities vary according to their quality or goodneſs.

N. B.

N. B. All bodies of what nature or kind soever being weighed in open air and ballanc'd by those whose specific gravity is greatest, I say those bodies whose specific gravity is least will weigh the heaviest in vacuo.

Thus if a piece of lead at the end of a nice ballance and a piece of cork at the other end are in equilibrio in the air, and thus placed under the receiver of an air-pump, as soon as the air begins to be exhausted, the equilibrium will begin to be destroyed, till at last when all the air is taken away, the cork will descend and shew itself really heavier than the lead. The reason of which is very evident from the laws of hydrostatics; for both bodies being weighed in air, each would lose the weight of an equal bulk of air, consequently the cork will lose a greater weight than the lead, because it is of greater bulk; and therefore when the air is taken away from both, the weight that is restored to the cork being greater than what is restored to the lead, will cause it to preponderate or weigh down the lead in vacuo.

Now let us take 14lb. of cork, which is 224 ounces, and $\frac{224}{240}$ is equal to .93 of a foot, which multiplied by 1728 gives 1612.8 solid inches in 14lb. of cork: And $\frac{224}{10130}$ is equal to .022112 of a solid foot, which multiplied by 1728 gives 38.2096 solid inches in 224 ounces of lead: Then 38.2096 subtracted from 1612.8 leaves 1574.5904 for the solid inches of air to be added to the cork; the weight of which quantity of air is thus found; As 1728 : 12 : : 1574.59 to 1.09346, equal to 1 oz. 1 $\frac{1}{2}$ drm. therefore the 14lb. of cork is 1 oz. 1 $\frac{1}{2}$ drm. heavier than the 14lb. of lead.——And for the same reason a

 pound

pound of feathers is heavier than a pound of lead; which may seem a paradox to some; but in reallity is no greater a paradox than the following one in finding the center of gravity, *viz.* That if a tapering piece of timber be nicely ballanced, and then cut in two at the place where the support for the ballance is, I say that if one piece is put into one scale and the other piece into the other scale, they will not ballance each other as they did before, but the tapering end will be the lightest.

In the foregoing example we make the cork and the lead to ballance each other in the fluid medium of air wherein neither could swim, now in the following example we will choose two bodies that will not swim in water, and then see how much the lightest body is heavier than the heaviest in air: And for this purpose we shall choose brazil-wood whose specific gravity is 1031, and lead whose specific gravity is 10130.

Now suppose we take 15lb. of each, that is 240 ounces, then 240 divided by 1031 gives .2327 part of a solid foot, which multiplied by 1728 gives 402.1 solid inches in 15lb. of brazil-wood: Again, 240 divided by 10130 gives .02369 part of a solid foot, which multiplied by 1728 gives 40.936 solid inches in 15lb. of lead. Now these different bulks must remove different bulks of water, and consequently that of the greatest bulk will meet with the greatest resistance from the water as it endeavours to sink; and therefore the equilibrium will be destroyed; now to find how much the equilibrium will be destroyed let us find the difference of their bulks, which is 361.164 solid inches equal to the quantity of water displaced by the brazil-wood more than

than is displaced by the lead. Now the weight of the water is found thus; As 1728 : 1000 : : 361.164 : 13lb. 1 oz. for the difference of their weights in water; that is, the brazil wood would require 13lb. 1 oz. to be added to it, to make a ballance with the lead; consequently if the ballance was first made in water and then weighed in air afterwards, the 13lb. 1 oz must have been added to the lead to make it ballance the brazil-wood.

CASE IV. *To determine the goodness of metals, &c.*

RULE. 1st Multiply the difference of the specific gravities of the compound and lighter ingredient, by the product of the specific gravity of the heaviest ingredient and absolute weight of the compound, for a dividend; 2d. Multiply the difference of the specific gravities of the ingredients by the specific gravity of the compound, for a divisor; 3d. Divide the dividend by the divisor, and the quotient will be the weight of the heaviest ingredient in ounces.

I have a piece of metal whose specific gravity is 15840, absolute gravity (or weight) 80 ounces (avoirdupois;) what is the quantity of gold, and also the quantity of silver contained therein?

See the work in the next page.

18888
10528
―――――
8360
15840
―――――
334400
66880
41800
8360
―――――
132422400

15840
10528
―――――
5312

18888
80
―――――
1511040
5312
―――――
3022080
1511040
4533220
7555200
―――――
132422400)802664480(60.6139
794534400
―――――
813004800
794534400
―――――
184704000
132422400
―――――
522816000
397267200
―――――
1255488000
1191801600
―――――
63686400

Silver 60.172 } Answ.
Gold 19.828 }
―――――
Compound 80.000

I have

I have 140 oz. of a mixture of copper and ſilver, whoſe ſpecific gravity is 9640; what quantity of ſilver is there in it?

```
   10528        9640          10528
    8796        8796            140
   -----        ----        -------
    1732         844         421120
    9640                     10528
   -----                    -------
    69280                   1473920
  10392                         844
 15588                      -------
 --------                   5895680
 16696480                  5895680
                          11791360
                         ----------
          16696480)1243988480(74.5
                   116875360
                   ---------
                    75234880
                    66785920
                    --------
                    84489600
                    83482400
                    --------
                     1007200
```

Anſw. { 74.5 copper.
{ 65.5 ſilver.

140.0

Quere, whether the aſſaying of gold, ſilver, &c. would not be much better performed upon hydroſtatical principles than by the preſent method by fire?

Note. That in buying a quantity of gold, ſilver, or any thing which is heavier than braſs (viz. the weights being of braſs,) we ſhould chooſe the lighteſt air, (i. e.) when the mercury in the barometer is loweſt; but in buying

precious ſtones, pearls, or any thing that is lighter than braſs, the beſt time to do it is when the air is heavieſt and moſt buoyant, viz. when the quickſilver ſtands higheſt in the barometer: But in ſelling gold or jewels, the contrary rules are to be obſerved in regard to the gravity of the air.

CASE V. *The ſolidity of any timber being given, to find how far it will ſink.*

RULE. Divide the ſpecific gravity of the timber by the ſpecific gravity of the water, multiply the quotient by the depth of the timber, and that gives the inches under water.

How many inches will a cubic foot of aſh or deal ſink in common water?

Aſh.		Deal.	
1000)838.000(.838		1000)657(.657	
12		12	
10.056	inches.	7.884 under	} water.
	Anſwer.	4.116 above	

CASE VI. *The ſolidity of any timber being given, to find how much it will carry.*

RULE. Subtract the ſpecific gravity of the timber from the ſpecific gravity of the water, the remainder is the number of ounces that one ſolid foot will carry.

How much weight is juſt neceſſary to ſink a cubic foot of deal in ſea water?

See the work in the next page.

```
      1030
       657
      ———  lb.
   16)373(23·313
      32   Answer.
      ——
       53
       48
       ——
        50
        48
        ——
         20
         16
         ——
          40
          48
          ——
```

How much weight will a raft made of 12 pieces of yellow deal carry (in sea water) if each piece is a foot square and 20 feet long?

```
      1030                 12
       657                 20
      ————                ————
       373                240
       240
    ———————
     14920
     746
    ———————  lb.
16)89520(5595  will just sink the deal under
    80              water, and keep the weight
    ———             compleatly above water.
     95
     80
     ———
     152
     144
     ———
       80
       80
     ———
```

Now

Now ſuppoſe it was loaden with braſs, whoſe ſpecific gravity 8104 being divided by 16, gives 506.5 pounds; and the pounds in a ſolid foot of ſea water is 64.4 pounds.

```
          5595
          64.4
        ------
         22380
        22380
       33570
      --------
506.5)360318.0 (711.3
       35455    5595
      -------  ------
         5768  6306.3  viz. it will prevent
         5065           6306.3 lb. of braſs
        ------          from ſinking, and
          7030          will juſt keep it ſuſ-
          5065          pen:ed in the water.
         ------
          19650
          15195
         ------
           4455
```

In the ſame manner is found by the comparative weight of the bodies, how much of any heavy quality or matter may be ſupported by the water.

How

How many inches will an aſh beam ſink in common water that is 14 by 11, and 18 feet long, when it has 200 lb. upon it?

```
       14
       11
     ----
      154
       18
     ----
     1232
     154
    -----
144)2772(19.25 ſolid feet.
    144
    ---
    1332
    1296
    ----
     360
     288
     ---
      720
      720
      ---
```

```
  1000
   838
  ----
   162 ounces that one
       ſolid foot will carry.
```

```
       19.25
         162
      ------
        3850
      11550
      1925
    --------
     3118.50
```

See the remainder of the work in the next page.

```
         lb.
16)3118.50(194.9
   16
   ---
   151
   144
   ---
    78
    64
   ---
    145
    144
   ---
     1
```

that is, it will but just carry 194.9 lb. and 195 lb. will sink it.

Now seeing that it will not carry 200 lb. as the question requires, let us suppose that 190 lb. was put upon it, then the operation will be as follows:

```
      lb.    In.     lb.
   As 62.5 : 1728 : : 190
                190
            -------
             155520
             1728
            -------  In.
   62.5)328320.0(5253
        3125
        ----
         1582
         1250
         ----
          3320
          3125
          ----
           1950
           1875
           ----
             75
```

which divided by the square inches of the surface gives the answer requir'd: See the work to each side uppermost.

Broadest

Broadest side uppermost.

```
                 28
                 12
               ----
                216
                 14
               ----
                864
               216
               ----
Surface        3024)5253(1.737
                     3024
                     -----
                     22290
                     21168
                     -----
                      11220
                       9072
                      -----
                      21480
                      21168
                      -----
                        312

1000)838(.838
          11
      ------
Inches  9.218 that it sinks naturally.
        1.737 sinks with 190lb. upon it.
      ------
       10.955 sinks in all ; so that there
is but   .045 out of water.
      ------
       11.000 depth.
      ------
```

Narrowest

Narroweſt ſide uppermoſt.

```
              18
              12
             ----
             216
              11
             ----
Surface 2376)5253(2.21
             4752
             ----
              5010
              4752
              ----
               2580
               2376
               ----
                204

1000)838(.838
          14
        ----
        3352
        838
        ----
11,732 ſinks naturally.
 2.210 ſinks more with the weight.
------
13.942 ſinks in all.
  .058 out of the water.
------
14.000 depth.
```

We might now proceed to ſhew the principles upon which bodies that are heavier than water are made to ſwim; but as this part of the hydroſtatics is upon a principle different from that of ſpecific gravity, we ſhall omit ſaying any thing on that head, unleſs it ſhould be deſired in a ſecond edition.

AN

AN INTRODUCTION TO PRACTICAL MUSIC.

IN practical music you'll observe there are but seven degrees of tones in the gamut, C D E F G A B; and if we proceed further it can be but by repeating the first note, and so on, so that the same notes repeated are called octaves: Thus the two notes that create the octave, are in main but one, and serve as limits or bounds to all the intervals, since all the notes in the gamut are included in an octave; 5 of which are whole tones, and the other 2 half tones, consequently making 12 half tones in the whole.

But the writers on speculative music, not taking the ear to be a competent judge in a case so nice, have chosen to distinguish the 12 half notes, not by equal intervals but by due proportions. And although we have these successive proportions for each semitone, it hath been found necessary in the construction of organs, flutes, hautboys, bassoons, &c. so to order the twelve intervals or semitones that their sounds shall be in one and the same proportion thro' the octave; and tho' each tone be rather bearing from its mathematical measure, yet the difference is so nice that very few ears can distinguish it: Therefore, we will follow the practical method of esteeming all the half tones equal one with another.

N. B. Where I could find inſtances of inſtruction from other modern publications that were plain and eaſy, I have taken the liberty of inſerting them along with what other alterations and additions I thought was convenient for the learner.

Note. I have omitted the gamuts for the ſeveral ſorts of inſtruments in general uſe, as numbers of each ſort abound; and have inſerted one general gamut of the notes which may ſerve for all.

The following is the Gamut of natural-notes, whether vocal or inſtrumental.

GABCDEFGAB CDEFGABCDEFG

The firſt thing to be obſerved is to learn the names of the notes upon the lines and ſpaces, by heart. Alſo due regard muſt be had to the cliffs ſet at the beginning of the five lines, which are an inlet to the knowledge of the notes; for if a note be placed on any part of the five lines, you cannot call it any thing till there is one or other of the cliffs ſet at the beginning: For which reaſon the lines of the gamut are divided into ſeparate fives, (or ſtaves,) expreſſing the different parts of muſic. For inſtance, the treble or uppermoſt five lines has G for its cliff, ſet at the beginning on the ſecond ine of the ſtave from the bottom. The lower-

most part or bass, has F for its cliff, and is commonly placed on the fourth line from the bottom. But there is another sort of cliff that comes between the treble and the bass, call'd the tenor cliff, thus ( ), and which may be set on any of the four lowest lines; it being C on that line whereon it is put.

The gamut may be repeated as well ascending as descending, and by different notes; but the distance from one note to the other must also be observed, and this only in ascending. The certain intervals are as follows.

From G to A is a whole tone, from A to B is a whole tone, from B to C is half a tone, from C to D is a whole tone, from D to E is a whole tone, from E to F is half a tone, from F to G is a whole tone, and so on with never so many notes, which must ascend in the same proportion of sound as the first eight notes do; as every note bears the same union of sound with its octave, whether it be an eighth above or below.

I shall instance an example or two of an octave: But for fear you should not tune the notes exactly, you ought to get the assistance of a person skill'd in music, and let him sing or play your eight notes with you, till you can do them without him.

Example.

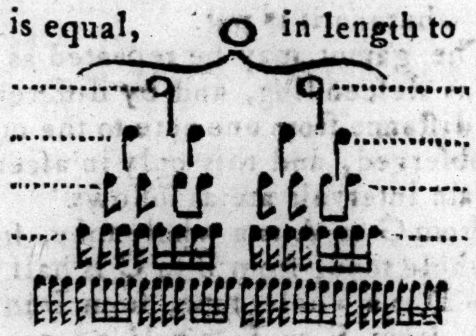
Example.

The next thing tow'rds the attainment of the knowledge of music, is to be well acquainted with the different measures of notes, which are under the denomination of time.

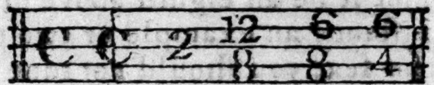
Of Time, or the length of notes.

A Semibreve is equal, in length to

Minims - or
Crotchets - or
Quavers - or
Semiquavers or
Demisemi-quavers



There are two sorts of time, common and triple. Common time is known by some of the following marks or characters.

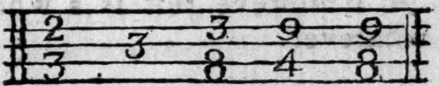


The first of these marks denotes the slowest kind of movement, and contains a semibreve (or as many other notes as are equal to its length) in a bar, and must be held as long as you can distinctly tell 1, 2, 3, 4. The second denotes a movement somewhat faster than the former, and contains also a semibreve in a bar. The third denotes a brisk movement, and contains but one minim, or two crotchets, &c. in a bar. The fourth mark contains twelve quavers (or

(or notes to their value) in a bar; the fiſth ſix quavers in a bar, and the laſt ſix crotchets in a bar. The three laſt characters are fixed to jiggs, &c.

In a ſlow common time you muſt divide the bar into four equal parts, telling 1, 2, 3, 4, diſtinctly, putting your hand or foot down when you tell one, which muſt be at the beginning of the bar, and lifting it up when you tell 3, which muſt be in the bar. In a quick ſort of common time, you may divide your bar into two equal parts, only putting your hand or foot down at the firſt half of the bar and lifting it up at the ſecond half; but you muſt be exact in moving up and down. Triple time, whether quick or ſlow, muſt be divided in three equal parts, telling 1, 2, with your hand down, and 3 with it up: In this ſort of time you muſt keep your hand up but half the time you keep it down. Obſerve thro' the whole the down beat at the firſt note in every bar.

The following are marks of triple time.



The firſt of theſe characters has three minims in a bar, and is the ſloweſt triple time in uſe. The ſecond contains three crotchets in a bar, and is fixed to minuets, and play'd quicker than the former. The third contains three quavers in a bar, and is the quickeſt. The fourth contains nine crotchets; the laſt nine quavers. Theſe laſt are rarely made uſe of, and then to jiggs.

 Ob-

Obſerve a diminutive note or grace is ſet before a real note, and is only meant to prepare that real note, and not reckoned into the time. Alſo when you find the figure 3 ſet over three ty'd notes, it ſignifies they are to be play'd and reckon'd in the time of two of thoſe notes.

Note. A point or dot added to any note, whether minim, crotchet, &c. makes it half as long again; and muſt always be put on the right ſide of the note.

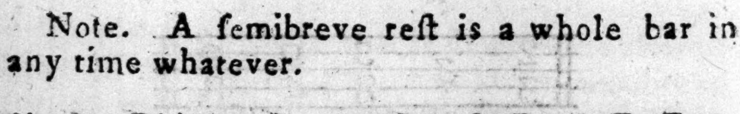
EXAMPLE of RESTS.

Semibreve. Min. Crot. Quav. Semiqua.

2 Bars 4 Bars 8 Bars 16 Bars 24 Bars.

Note. A ſemibreve reſt is a whole bar in any time whatever.

Sin. Bar. Dble Bar. Repeats Directs. Pauſe. Da Capo



A ſingle bar ſerves to divide the time according to its different meaſures, whether common or triple. A double bar ſerves to divide every ſtrain or part of a ſong or leſſon. A repeat ſignifies

nifies that such a part of a song, &c. must be performed over again from the note over (or before) which it is put. A direct is put at the end of a stave, and serves to direct to the place of the first note in the next stave. A pause signifies that the note over which it is placed must be held out somewhat longer than the usual time.—The same mark also denotes the end of a tune. Da Capo signifies the tune does not end there, but must be begun again, and play'd till you come to the mark 𝄐 aforemention'd to denote the end of the tune.

Of Flats and Sharps, &c.

These characters are very significant in music, and must be particularly regarded; viz. (♭) or Flat; (♯) or Sharp; (♮) or Natural. If a flat be placed before any note it denotes that such note (and all the following in the same bar on the same line or space except mark'd to the contrary) must be sung or play'd half a note lower than its natural pitch. The sharp is of a contrary nature; for whereas a flat takes away a semitone, or half a note, from the sound of the note before which it is set, the sharp adds a semitone to whatever note it is set before. For example, if a flat (or flats) be fixt at the beginning of any of the five lines, it not only affects every note on such line or space but also all the notes of that denomination thro' the whole movement. The same is likewise to be observed of the sharps. A natural serves to reduce any note made flat or sharp by the governing flats or sharps placed at the beginning, to its primitive sound, as it stands in the gamut.

I shall

I ſhall next inſtance ſome ſelect pieces for leſſons, both for the voice and inſtrument: But you muſt be particular in obſerving the time, as beforemention'd; for no muſic can be agreeable to the performer unleſs he makes himſelf maſter of it; neither is it poſſible for ſeveral performers to agree exactly together without it.

N. B. The following eight pages of copper-plates containeth ſix leſſons for different inſtruments, and three ſongs. The ſix leſſons are adapted to the under-mentioned inſtruments;

viz.
- 1ſt. For two French Horns.
- 2d. For the Guittar.
- 3d. For the German Flute.
- 4th. For ditto.
- 5th. For ditto.
- 6th. For the Violin.

COLLIN and CLORIS.

Continued.


just was her motion, how sweet was her
song: But when the Youth told her his
passionate flame, She allow'd him to
fancy her heart felt the sa . . . .
. . . . . me. She allow'd him to fancy her
Sym.
heart felt the same.

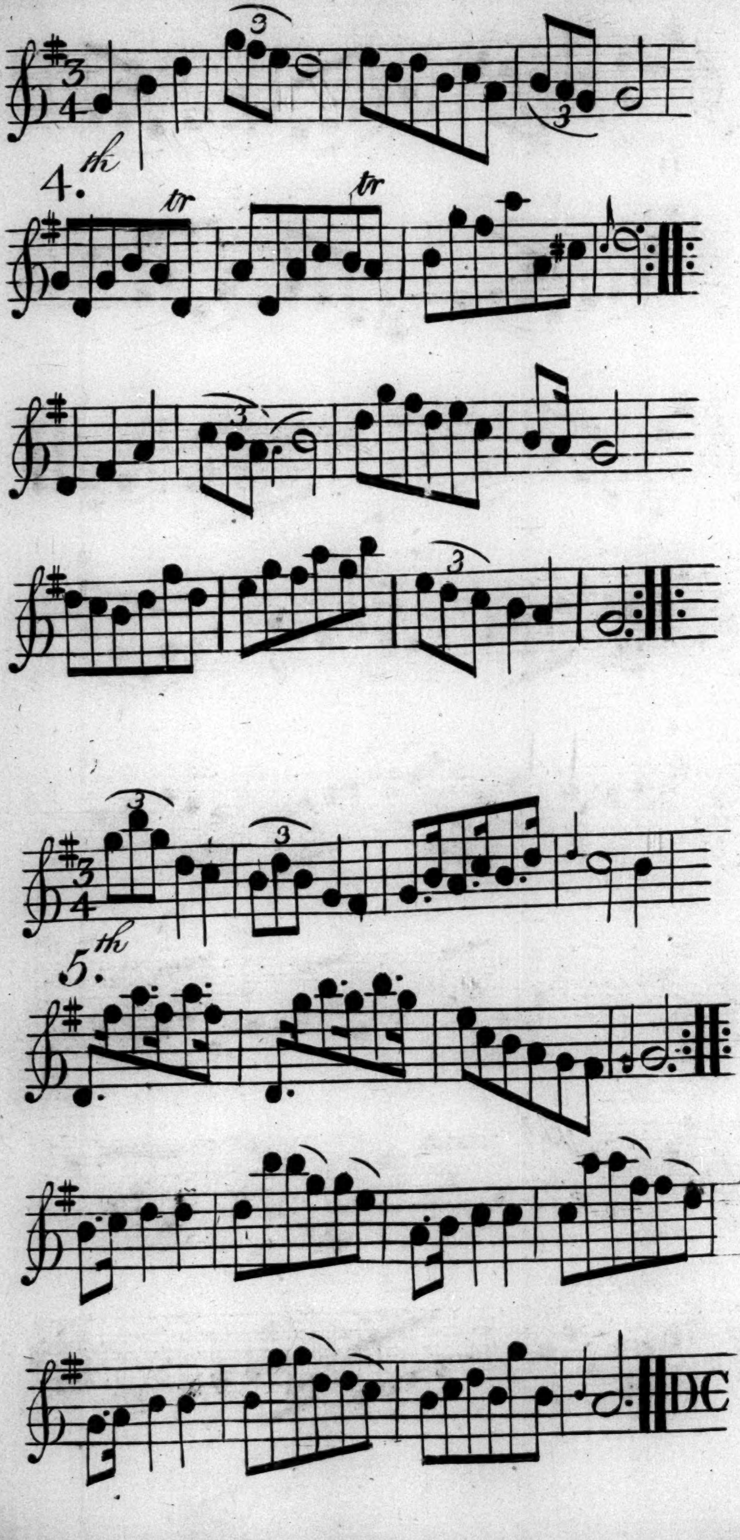
SONG.

See remainder of the words in page 305.

CHARMING CLOE.


1st
3


2.nd
tr
3.rd
tr
3



4.th
tr
tr
5.th
DC


6.th
tr
tr
tr
tr
tr
tr tr
tr
tr

The remainder of the words to the first Song.

II.

Could I but once on him prevail,
To mingle with his joys my ſmart;
That he may feel what now I ail;
But I'm too young to ſhew ſuch art.

III.

Attractive Cupid be my care,
And look with pity on my pain:
Oh! break the chain that now I ware,
Or bind Ominton in the ſame.

IV.

Haſte to thy mother tell my grief,
To help an harmleſs injur'd maid;
That he may quickly ſend relief,
And ſave an heart that is betray'd.

Remainder of the words to Charming Cloe.

II.

Strephon, mov'd by lawful paſſion;
For no favours rudely ſues;
All his flame is out of faſhion,
Ancient honour for him wooes.
Love for love 's the ſwain's ambition;
But if that is deem'd too great,
Pity, pity his condition;
Say at leaſt, you do not hate.

III.

Should you, fonder of a rover,
Practis'd in the art of guile;
Slight ſo true and kind a lover,
Cloe, might not Strephon ſmile?

Yes

Yes; well pleas'd at thy undoing,
Vulgar lovers might upbraid;
Strephon, confcious of thy ruin,
Soon would be a filent fhade.

Remainder of the words to Collin and Cloris.

II.

With ardour he preft her to think him fincere,
But alas! fhe redoubl'd each hope and each fear;
She wou'd not deny nor fhe wou'd not approve,
And fhe neither refus'd him nor gave him her love.
Now cheer'd with compliance, now froze by difdain,
He languifh'd for freedom, but languifh'd in vain;
Till Thyrfis who pity'd fo helplefs a flave
Eas'd his heart of the pain by the council he gave.

III.

Forfake her, faid he, and reject her awhile,
If fhe loves you fhe foon will return with a fmile;
You may judge of her paffion by abfence alone,
And by abfence you'll conquer her heart or your own.
This advice he purfu'd, but the remedy prov'd
Too fatal alas! to the fair one he lov'd;
Which cur'd his own paffion, and left her in vain
To figh for an heart fhe cou'd never regain.

To

To make the Trillo or ſhake on the voice.

The trill or ſhake, (mark'd t, or tr.) is the chief grace in ſinging, and has a fine effect when well performed, To learn this you muſt raiſe your voice on one ſyllable, the diſtance of a whole tone, as in the following example. Firſt move your voice ſlowly, then faſter, by degrees, and it will come to you with a little practice; but you muſt be ſure to let the two notes be both heard diſtinctly.

The trill or ſhake is to be made on all deſcending prickt crotchets, and alſo when the note before it is in the ſame line or ſpace with it; likewiſe generally before a cloſe, either in the middle or at the end of a ſong.

Note. If you wou'd practice ſinging, the application of the inſtrument will be of the utmoſt aſſiſtance to the attaining to a proficiency therein.

Of the Keys in Music.

The word key is adapted to one note choſen as principal to compoſe a piece of muſic in, it being the ending note, ſo called the fundamental or key-note, as it determines the certain intervals, wherein all the tones or whole notes, and ſemitones or half notes, which ought to follow each other from the key-note to its octave, take place.

There is properly but two keys in muſic, one flat, the other ſharp; but by the help of flats and

and ſharps they have been varied to the number of twenty-four in the whole; one half of which are ſharp keys, and the other half flat keys: Thoſe moſtly in uſe are as follow.

A SCALE of SHARP KEYS.

C, natural ſharp third.

D, with a ſharp third.

E flat, with a ſharp 3d.

E, with a ſharp third.

F, with a ſharp third.

G, with a ſharp third.

A, with a ſharp third.

B flat, with a ſharp 3d.

A Scale of Flat Keys.

A, natural flat third.

B, with a flat third.

C, with a flat third.

D, with a flat third.

E, with a flat third.

F, with a flat third.

F ſharp, with a flat 3d.

G, with a flat third.

Now for the better diſtinction of the whole tones and half tones in the keys, you muſt obſerve, that in the gamut, the half notes are naturally found betwixt E and F, and between B and C, whereas a *whole* tone is found betwixt all

the other notes; and there being but ſeven different denominations, C, D, E, F, G, A, B, ſo there is but 7 different degrees of notes in the whole gamut (as before obſerv'd,) for all eights bear the ſame ſound as well as name: Likewiſe the flats and ſharps ſet at the beginning of a tune have the ſame effect on all their octaves above or below. As you may take whatever note you think proper for a key-note, provided you give a progreſſion to its octave, like uuto that of C, if the key be ſharp; then ſharps or flats are placed after the cliff, in order to increaſe or leſſen, a ſemitone, thoſe intervals that might hinder that conformity: And as the half tones lie in particular places in the key, I have dotted the ſemitones included in the octave of every key in the afore-mention'd ſcales, as they are found in the natural keys with no ſharp or flat at the beginning, to which order and progreſſion all the other keys are modelled. Whence it will be found no very difficult matter to order the flats or ſharps at the beginning of a tune according to the conformity of the key, whether it be a flat key or a ſharp one.

For example, if we take D for a key-note, and would make its key agreeable to that of C, we muſt begin with the key-note aſcending; and as we reckon on, we ſhall find that both F and C muſt be ſharp'd, to make it conformable to the key of C.

The flat key of A may ſerve to illuſtrate all flat keys.

N. B. A key is known to be flat or ſharp not by what flats or ſharps are ſet at the beginning of a tune, but by the third above the laſt note, or key-note; for if a third contains two whole tones

tones it is a ſharp key, but if only a tone and a half 'tis a flat key. Always name your key in reference to the baſs.

Of Tranſpoſition.

To tranſpoſe a ſong or leſſon that is too high, or too low, or in an improper key for a voice or inſtrument, you muſt firſt ſee what compaſs the tune requires; that is, how high and how low it goes, and accordingly take your meaſure. The laſt note of a tune, as before obſerved, tells you what key it is, whether A, B, C, D, &c. Therefore, ſuppoſe for example, you had a ſong or leſſon in the ſharp key of E, and you wanted it tranſpoſed into G, which is a third higher; look in the aforegoing ſcale for G with a ſharp third, where you'll find one ſharp, and that upon F; ſo placing the ſharp as at the beginning, and writing each note a third higher than it is in the copy, you have the tune right, in the deſired key.

You may tranſpoſe into any of the keys in the reſpective ſcale, obſerving the diſtance or number of notes from the key-note of your copy to the firſt note of the key you tranſpoſe into, and putting the ſame number of flats or ſharps at the beginning as in the ſcale.

Note. Whatever accidental flat, ſharp, or natural is met with from the beginning of a tune to the end, the ſame muſt be obſerved in that part of the tranſcript by ſome one or other of thoſe powers: Thus, when a natural is met with (this way) you muſt have regard to the effect it has on the note it ſtands before, whether it be of a flat or ſharp quality, and in ſuch

caſe you muſt ſupply its place with either flat or ſharp, according to the effect it has in the copy. Obſerve the ſame order with the other powers; knowing a flat to one note is a ſharp to the note below it, except in a caſe where a natural half tone lies.

Now as there are ſeveral other places beſide what are mention'd in the aforeſaid ſcales which either of the keys may be tranſpoſed into upon occaſion, the reaſon aſcribed is, that every half note in the gamut ſhould have its fundamental or key-note.

The following is a SCALE *of all the* KEYS.

In this ſcale you'll obſerve ſtraight lines interſecting the common ſtave or five lines, and letters ſet both above and under the ſtave, ſignifying the keys to which the diviſions belong; each letter above the ſtave denotes a ſharp key anſwering to whatever flats or ſharps are under it, and the letter underneath denotes a flat key agreeing to the ſame flats or ſharps: As in the natural keys, C above the ſtave denotes a ſharp key, and A (a flat third below C) underneath the ſtave, ſignifies a flat key; and ſo with the reſt.

THE

THE CONSTRUCTION OF ORGAN-PIPES, SPINETS, &c.

WHAT is to be obſerved in the conſtruction of organ-pipes is the hollow part or cavity from whence the ſound proceeds; therefore in this caſe we have nothing more to do than to find the dimenſions of ſuch particular cavities that will ſound any given note in the octave. And firſt the half of each particular dimenſion of any pipe gives the octave above, and the double of each particular dimenſion of any pipe gives the octave below.

Alſo we have the ſame reaſon for muſical ſtrings, &c. For inſtance, ſuppoſe two ſtrings of equal thickneſs, and ſtretched with equal weights, and have their lengths as 1 to 2, viz. one ſtring twice the length of the other, they will ſound an octave to each other; and conſequently in like manner the ſounds of any other ſtrings will be in proportion to their diameters and the weights that ſtretch them.

Suppoſe it was required to find a mean ſemitone for the conſtruction of organ-pipes, &c. ſo that their ſounds with regard to gravity or acuteneſs may be in the ſame proportion which altogether ſhall compleat the octave. In order to

 this,

this, we know that the octave is known by this fraction $\frac{1}{2}$; and as there are 12 half notes in the octave, all melody and harmony are composed of these 12 notes; for the octave above or below are but the replicatives of the same sounds in an higher or lower tone. Therefore the following table is the measure of the distance to any particular half tone required in the octave; and they are also factors to find the dimensions of musical chords and other instruments.

TABLE.

	1.00000
1	.94388
2	.89090
3	.84090
4	.79370
5	.74915
6	.70710
7	.66742
8	.62996
9	.59460
10	.56123
11	.52973
12	.50000

To apply the factors in the table to the construction of organ-pipes.

Here you are to observe that the dimensions of the cavity of some one pipe must be given; and each particular dimension must be multiplied by the factor in the table, and the product gives the like dimension of that pipe that will sound the note belonging to that factor. Thus for

for example, ſuppoſe C the key-note, and the given dimenſions of the pipe to be 1.57 by 1.46, and 20.9 inches long. Now ſuppoſe I would have the dimenſions of that pipe to ſound G, viz. 7 half tones higher than the given pipe. Then 1.57 multiplied by .66742 gives 1.0478; 1.46 multiplied by .66742 gives .9744; and 20.9 multiplied by .66742 gives 13.9491; and thus you have the dimenſions of the pipe required.

The dimenſions for all the ſemitones in the above octave are as follow.

	Length	Breadth	Width
C	20.9000	1.5700	1.4600
C ſharp.	19.7271	1.4819	1.3701
D	18.6198	1.3987	1.3007
D ſharp.	17.5748	1.3202	1.2277
E	16.5883	1.2461	1.1588
F	15.6572	1.1762	1.0937
F ſharp.	14.7784	1.1101	1.0324
G	13.9491	1.0478	0.9744
G ſharp.	13.1662	0.9890	0.9197
A	12.4271	0.9335	0.8681
A ſharp.	11.7297	0.8811	0.8194
B	11.0713	0.8317	0.7734
C	10.4500	0.7850	0.7300

If you would have an octave below, multiply the dimenſions of any pipe by 2 and it gives the dimenſions of that pipe an octave below; and take ½ the dimenſions it gives an octave above: Proceed in this manner for double octaves either above or below. In

In conſtructing round pipes of a cylindrical form, the ſame rules muſt be obſerved as with thoſe which are ſquared, and the given length and diameter only made uſe of in the operations.

To apply the factors in the table in conſtructing of muſical ſtrings.

In this caſe, multiply the length of the given ſtring by any of the factors in the table, and the reſult is the length of the ſtring that will ſound the note belonging the factor made uſe of if the thickneſs and tenſion are the ſame.

What muſt be the length of that ſtring to ſound 4 half tones above another that is 40 inches long?

$$\begin{array}{r} \cdot 7937 \\ 40 \\ \hline \end{array}$$

Anſwer 31.7480 inches.

What muſt be the length of thoſe ſtrings to ſound the notes D, E, F, G, A, B, C, if the key-note C is 40 inches long?

·8909		35.636	for D.
·7937		31.748	for E.
·74915	multiplied	29.966	for F.
·66742	by 40	26.696	for G.
·5946	produces	23.784	for A.
·52973		21.189	for B.
·5000		20.000	for C.

Suppoſe

Suppoſe it is required to find the weight that any ſtring muſt be ſtretched with to ſound any particular note, by having given the weight requiſite to ſound the key-note.

RULE. Divide the given weight by the ſquare of the factor in the table, the quotient will give the weight required to ſound any note above the key-note, if the ſtrings are of equal lengths.

What muſt be the weight to ſtretch a ſtring 40 inches long that it may ſound an octave above the key-note, if the key-note is ſtretched with 7 pounds, and is 40 inches long alſo?

```
 .5000
 .5000
 ─────
.25)7.00(28 pounds, the weight required.
    50
    ───
    200
    200
    ───
```

From whence we find, that if the diameters and lengths of muſical ſtrings are equal, the times of the vibrations (and conſequently the note ſounded) will be inverſly as the ſquare roots of the weights which ſtretch them; for if the weights are as 1 to 4, whoſe ſquare roots are 1 and 2, then the times of vibration will be as 2 to 1, and conſequently they will ſound octaves to each other. Hence in conſtructing ſtringed inſtruments, as ſpinets, harpſichords, &c. a ſkilful artiſt will compound theſe proportions of the length, diameter, and tenſion of ſtrings, to very great advantage.

F I N I S.

ERRATA.

Page.	Line.	
4 —	10	*for* underneatth *read* underneath.
6 —	22	*for* number *read* numbers.
10 —	24	*for* 4790*64 *read* 4790*84, &c.
29 —	9	*for* .92916 *read* .02916.
35 —	8	*for* factots *read* factors.
50 —	30	*for* gains of each *read* gains or losses of each.
110 —	3	*for* exttact *read* extract.
118 —	5	*for* of tower *read* of the tower.
138 —	31	*for* term *read* terms.
147 —	26	*for* what *read* what is.
175 —	17	*for* $\frac{1}{3}$ *read* $\frac{1}{5}$.
237 —	12	*for* measure *read* go.
242 —	31	*for* one one *read* one.
273 —	2	*for* may *read* many.
290 —	29	*for* ine *read* line.
ibid. —	30	*for* line *read* most.

Index